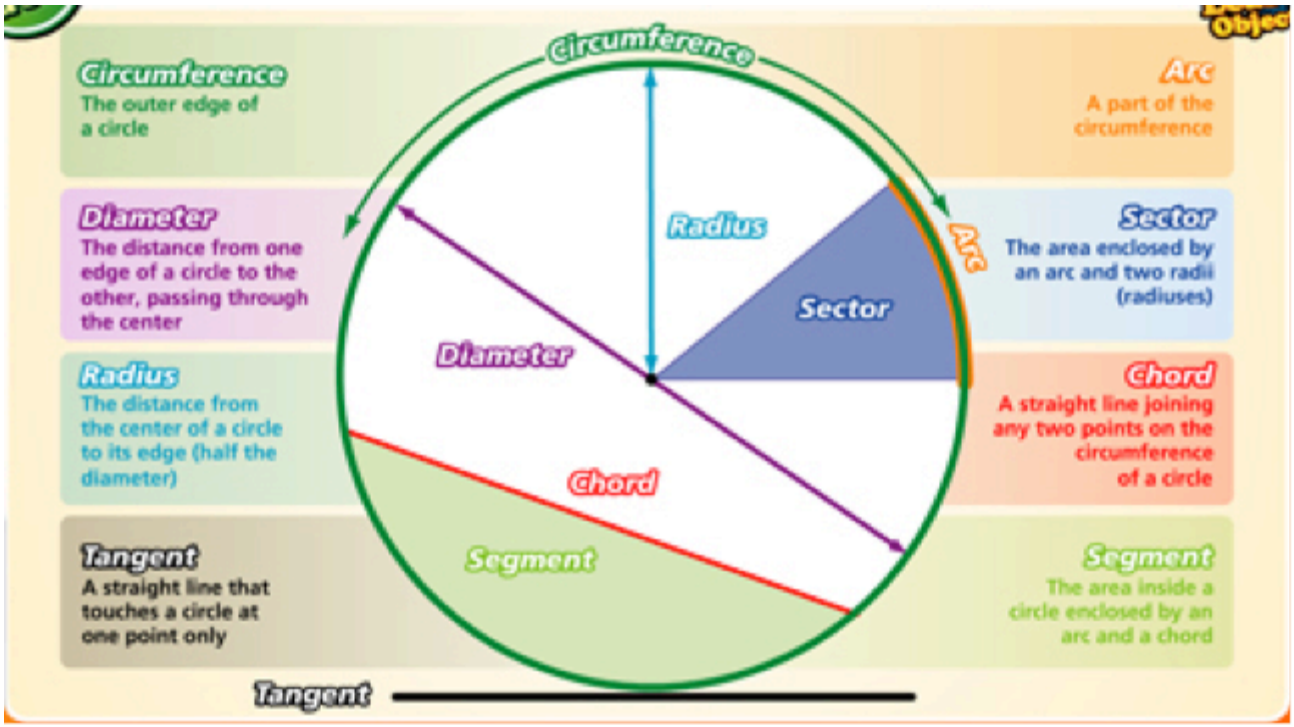
MATHEMATICS 9

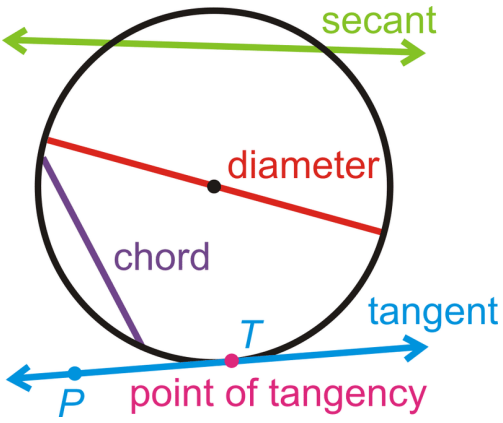
CIRCLE GEOMETRY

Section 8.1 – Properties of Tangents to a Circle



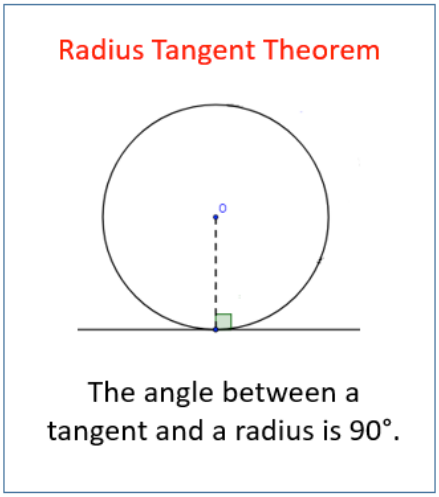
A **tangent** is a line that touches a circle at exactly one point.

The **point of tangency** is the point where the tangent touches the circle.



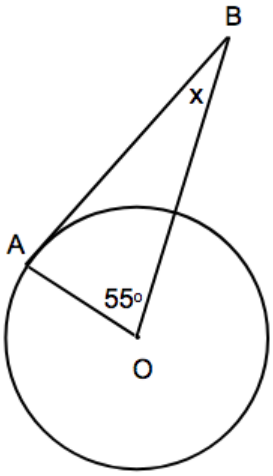
Tangent – Radius Property

"A tangent to a circle is perpendicular to the radius at the point of tangency."

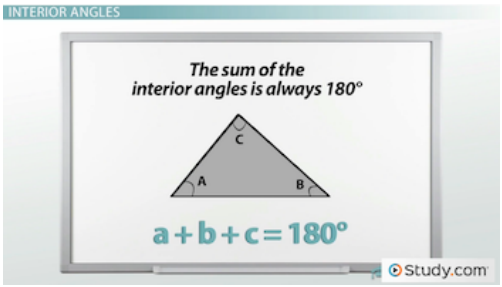


Example:

Point O is the center of the circle and AB is tangent to the circle.
In ΔOAB, $< AOB = 55°$. Determine the measure of $< OBA$.



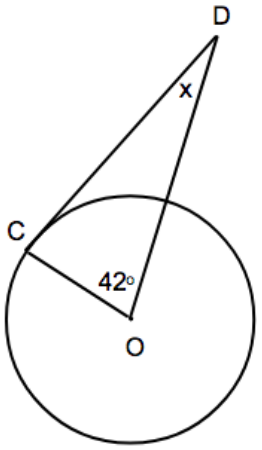
Recall,



Example:

Find the missing angle, x, in each diagram.

a)



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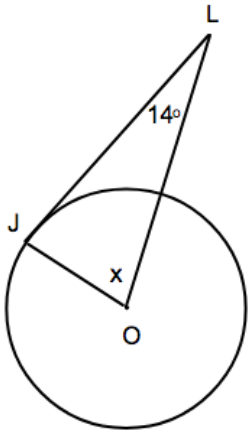
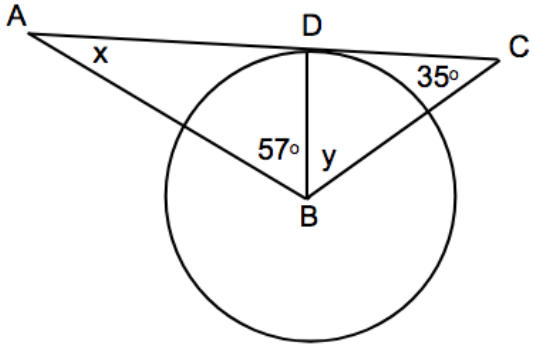


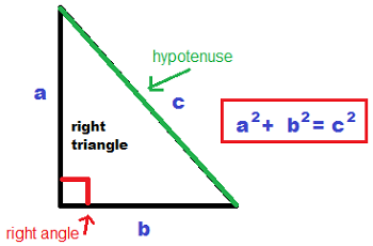
Example:

Find the missing angles, x and y, in the diagram. B is the center of the circle.



Using the Pythagorean Theorem in a Circle

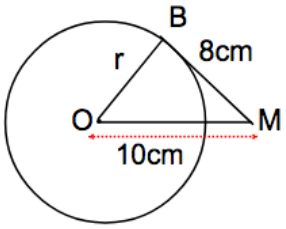
Remember that Pythagorean Theorem can be used to find a missing side in a right triangle: $a^2 + b^2 = c^2$

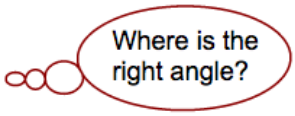


Example:

For each circle, find the length of the radius, r.

a)





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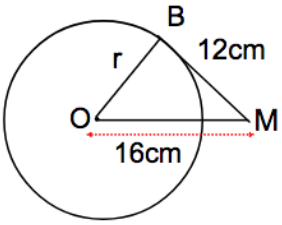
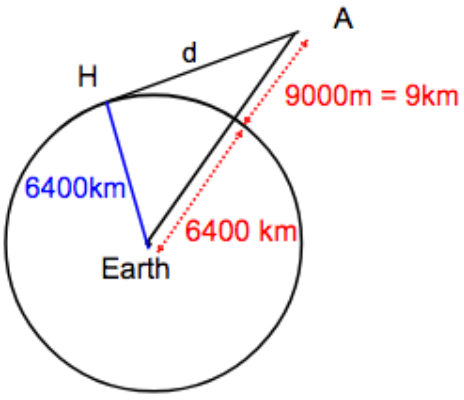


Example:

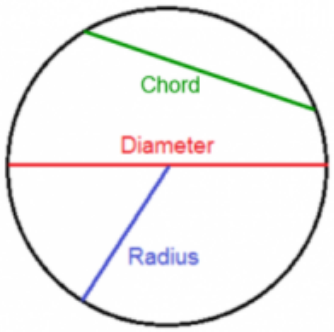
An airplane is cruising at an altitude of 9000m. A cross section of the earth is a circle with a radius approximately 6400km. A passenger wonders how far she is from a point H on the horizon she sees outside the window. Calculate the distance to the nearest kilometer.



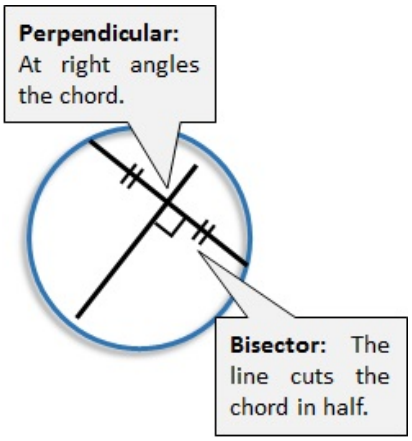
Section 8.2 – Properties of Chords in a Circle

A **chord** is a line segment that joins two points on a circle.

The **diameter** of a circle is a chord that passes through the center of the circle. It is the largest possible chord that can be drawn.



Perpendicular means there is a 90^0 angle.
Bisector means it is divided into 2 equal parts.



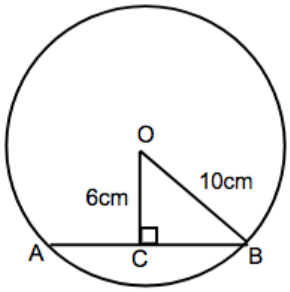
Perpendicular to Chord Property

"In any circle with center O, and chord AB, then:

- *If OC bisects AB, then OC is perpendicular to AB*
- *If OC is perpendicular to AB, then OC bisects AB, and AC = CB*
- *The perpendicular bisector of AB goes through the center O."*

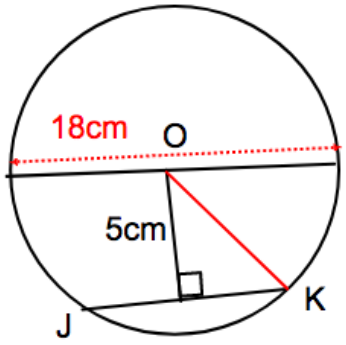
Example:

O is the center of the circle. Find the length of chord AB.



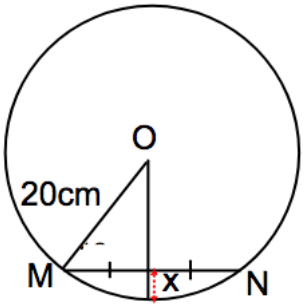
Example:

The diameter of a circle is 18cm. A chord JK is 5cm from the center. Find the length of the chord.



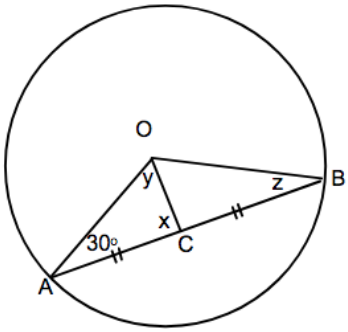
Example:

A chord MN is 24cm. The radius of a circle is 20cm. Determine the length of x in the diagram below.



Example:

Determine angles x, y and z.



Note:

Two radii

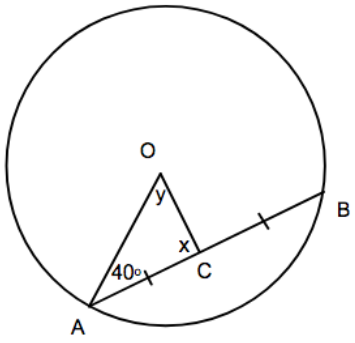
When the ends of a chord are joined to the centre of a circle, an isosceles triangle is formed.

The two angles marked are equal.

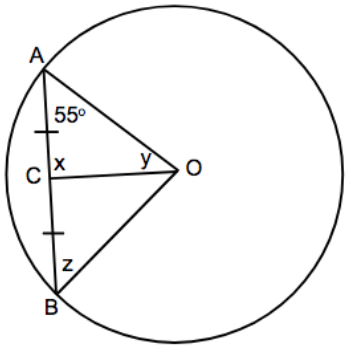
Example:

Find the missing angles in each circle.

a)

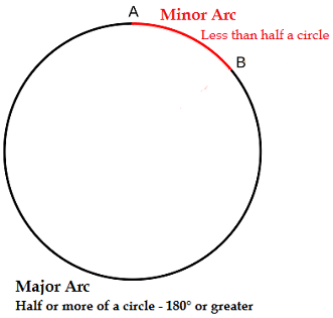


b)

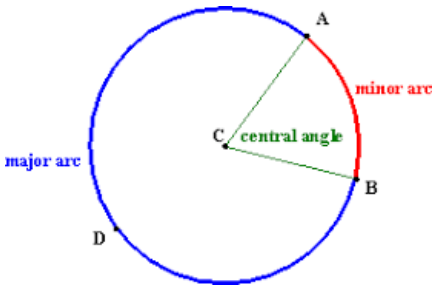


Section 8.3 – Properties of Angles in a Circle

An **arc** is a section of the circumference of a circle. There can be a **major arc** or a **minor arc**.

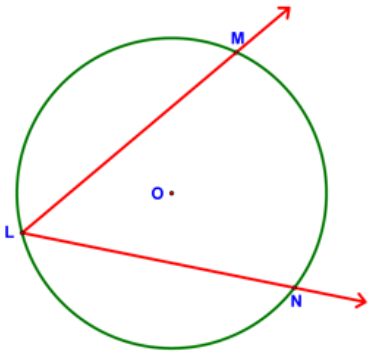


A **central angle** is the angle formed by joining the endpoints of an arc to the center of the circle.



< ACB is a central angle and it is ***subtended*** by the minor arc AB.

An **inscribed angle** is the angle formed by joining the endpoints of an arc to any point on the circumference of a circle.

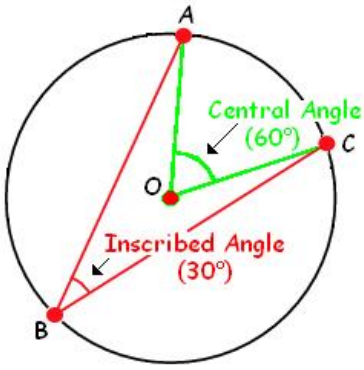


An *inscribed angle* is an angle whose vertex lies on a circle and whose sides contain chords of the circle (∠*MLN* is an inscribed angle).

< MLN is an inscribed angle and it is ***subtended*** by the minor arc MN.

Central Angle and Inscribed Angle Property

*"The measure of a central angle is **twice** the measure of an inscribed angle subtended by the same arc."*

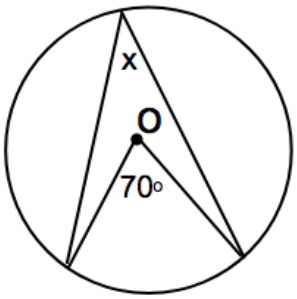


Central angles are double the measure of the inscribed angle that encloses the same arc ($\overset{\frown}{AC}$)

Example:

Determine the value of x in each circle.

a)



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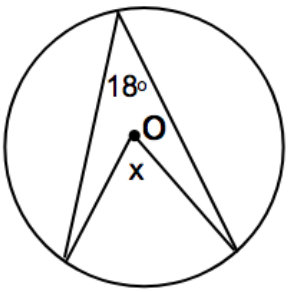
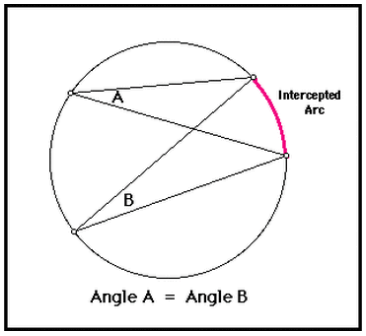
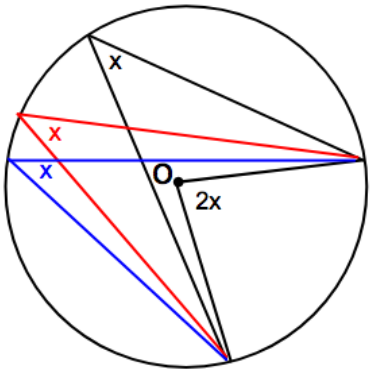


Inscribed Angles Property

*"Inscribed angles subtended by the same arc are **equal**."*

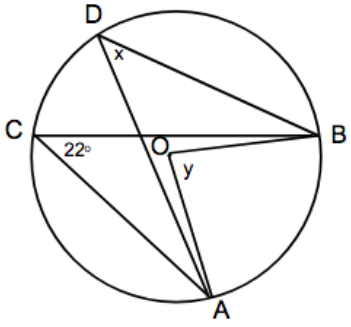




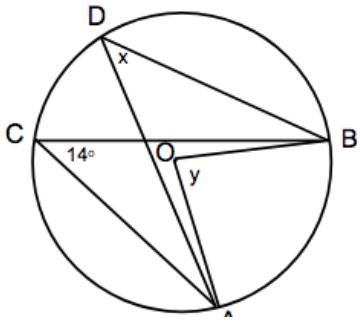
Example:

Determine the missing angles x and y in each circle.

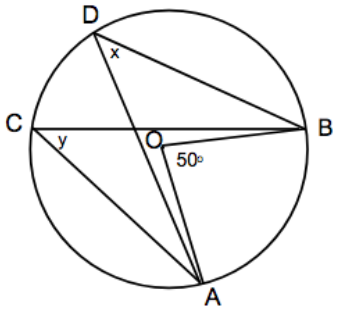
a)



b)

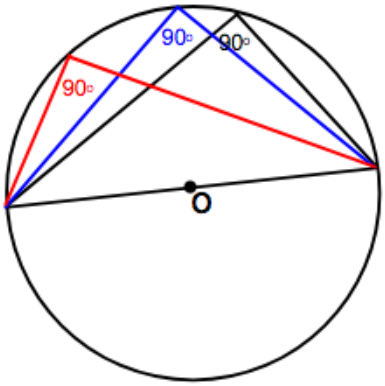


c)



Angles in a Semicircle Property

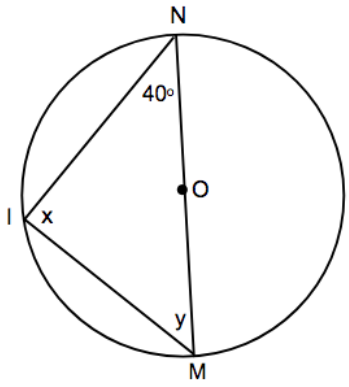
"Inscribed angles subtended by a semicircle (half the circle or the diameter) are right angles."



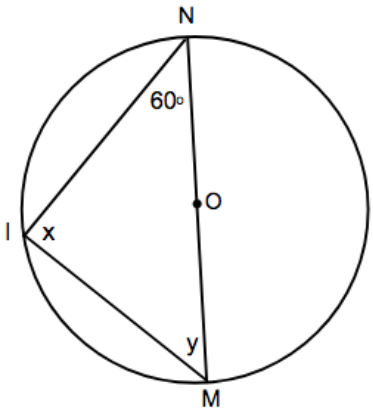
Example:

Determine the missing angles x and y in each circle.

a)

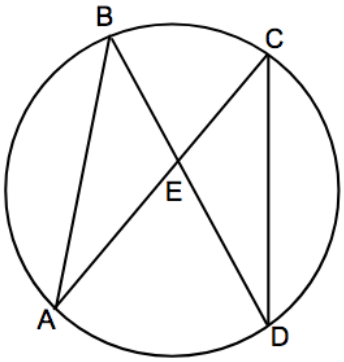


b)



Example:

Given $< B = (6x - 14)°$ and $< C = (4x + 2)°$.



a) Determine the value of x.

b) What is the measure of $< ABE$?