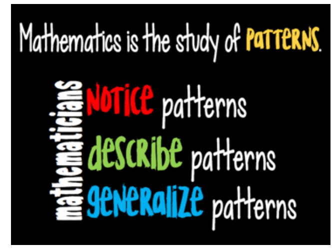
MATHEMATICS 9

LINEAR RELATIONSHIPS & **INEQUALITIES**

Section 4.1 – Writing Equations to Describe Patterns



In this unit we will be describing linear relations using words, equations, graphs, tables and pictures.

Let's consider the following example:

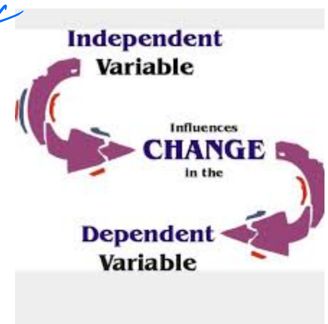
Luke wants to earn money this winter by shoveling driveways. He will get paid $9 an hour.

a) What two quantities are being compared in this problem?

↳ #, countable

Amount earned and hours worked

b) Which quantity is the independent variable and which is the dependent variable?



The **independent variable** is a variable whose value **is not** determined by the other variable. It does not depend on anything.

Independent variable: hours worked

The **dependent variable** is a variable whose value is determined by the other variable. It always depends on the independent variable.

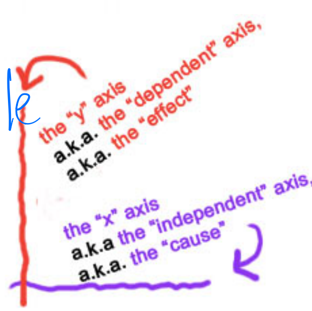
Dependent variable: amount earned

When trying to figure out which quantity is the independent and which is the dependent variable, complete the following sentence.

amount earned depends on the hours Luke works.

↑ **Dependent** ↑ **Independent**

"The dependent variable depends on the independent variable."



1 = x | 2 = y

y

(x, y)

x (independent variable)

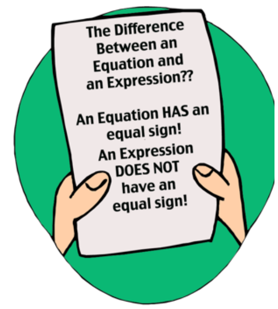
c) Complete the table of values.

Independent Variable → x   y ← Dependent Variable

Number of hours worked, h	Amount Earned, A
1	9
2	18
3	27
4	36
5	45

+9

d) Write an **expression** to represent the amount Luke earns per hour.



→ $9 \cdot h$ or $9h$ or $9 \times h$

e) Write an **equation** to represent the amount Luke will earn.

$A = 9h$

- Whenever the independent variable **increases by 1**, we can use the following strategy to find the equation.

x   y

Number of hours worked, h	Amount Earned, A
1	9
2	18
3	27
4	36
5	45

Find the difference in the dependent variable

Increased by 9
9
9
9

- The number that the dependent variable increases by should be the same!
- Use this number to multiply the independent variable by.

	Number of hours worked, h	Amount Earned, A
X9	1	9
X9	2	18
X9	3	27
X9	4	36
X9	5	45

9
9
9
9

Does $1 \times 9 = 9$? YES
Does $2 \times 9 = 18$? YES
etc
Therefore, the equation is:

A = 9h

f) Use this equation to determine how much Luke will make shoveling 11 hours in one week.

$A = 9h$

$h = 11$ $A = 9 \cdot 11 = \$99$

g) Use the equation to determine how many hours Luke will need to shovel to earn $72.

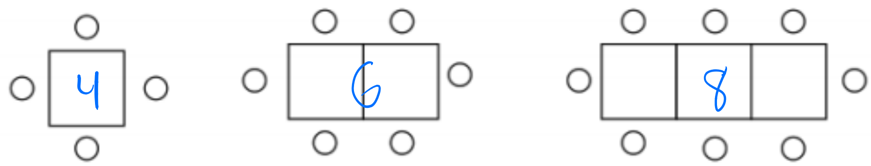
$A = 9h$ $\therefore h = 8$ hours

$A = 72$ $72 = 9h$

$72 = 9 \cdot$ ___

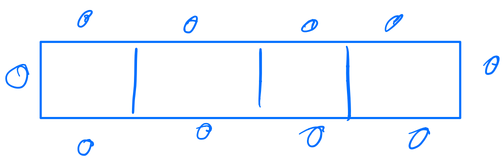
Example:

A banquet hall has small square tables that seat 1 person on each side. The tables can be pushed together to form longer tables, as shown below.

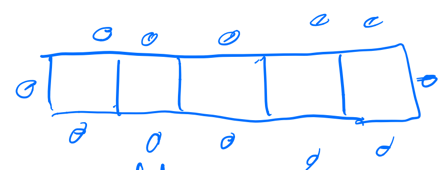


1 table 2 tables 3 tables

a) Sketch the next two tables arrangements in the pattern.



4 tables



5 tables

b) What stays the same in each table arrangement?

- pattern
- 1st table stays the same there is alway 4 original people

What changes?

- Amount of people increases by 2 for every new table

Description in words.

c) What are the independent and dependent variables?

\# of people depends on # of tables

Dependent Independent

"The dependent variable depends on the independent variable."

d) Complete the table of values.

Number of Tables t	Number of People p
1	4
2	6
3	8
4	10
5	12

Verify

$p = 2t + 2$

$p = 2(4) + 2 = 10$ ✓

$p = 2(2) + 2 = 6$ ✓

e) What expression can be used to represent the number of people?

$2t + 2$

f) Write an equation, using the variables, **t**, for the number of tables and **p**, for the number of people, for this problem.

$p = 2t + 2$

g) Use the equation to determine the number of people at:

i) 6 tables?

$p = 2(6) + 2$

$p = 12 + 2 = 14$ people

ii) 10 tables?

$p = 2(10) + 2$

$p = 20 + 2$

$p = 22$ people

iii) 25 tables?

$p = 2(25) + 2$

$p = 50 + 2$

$p = 52$ people

h) What number of tables are needed to seat 30 people?

$p = 2t + 2$

$30 = 2\,\underline{?} + 2$

$30 = 2(14) + 2$

$30 = 30$

$t = 14$

$p = 2t + 2$

$\frac{p-2}{2} = \frac{2t}{2}$

$t = \frac{p-2}{2} = \frac{30-2}{2} = \frac{28}{2} = 14$

If I don't need to know yet

Example:

Write an equation for each table below. Verify your answer by substituting values from the table.

a).

n	p
1	3
2	7
3	11
4	15
5	19

+4

b).

f	b
1	4
2	7
3	10
4	13
5	16

+3

Equation:

a) when $n = 1$

$4(1) = 4 - 1$

$\boxed{4n - 1 = p}$

Equation:

b) $3f = 3(1) = 3$

$\boxed{3f + 1 = b}$

Example:

Jiffy Cabs charges a fixed rate of \$3.60 plus \$1.50 per kilometer driven.

a) Write an expression for the cost of a ride in the cab.

$3.60 + 1.50k$

b) Write an equation for the total cost of a cab ride.

$C = 3.60 + 1.50k$

c) What is the cost of a 12 km cab ride?

$C = 3.60 + 1.50k$

$C = 3.60 + 1.50(12)$

$C = \$21.60$

Example:

An airplane is cruising at a height of 9000 m. The table below shows the height of the plane every minute after it begins to descend to land.

Time, t (Minutes)	Height, h (meters)
0	9000
1	8700
2	8400
3	8100
4	7800

a) Write an equation that relates the height of the plane to the time.

	Time, t (Minutes)	Height, h (meters)	
-300 x	0	9000	-300
-300 x	1	8700	-300
-300 x	2	8400	-300
-300 x	3	8100	-300
	4	7800	

The difference decreased by 300

$9000 - 300t$

$-300 \times 0 + \_\_\_\_ = 9000?$
$-300 \times 1 + \_\_\_\_ = 8700?$
$-300 \times 2 + \_\_\_\_ = 8400?$ etc

The missing number is 9000.

Equation:

$h = 9000 - 300t$

b) What is the height of the plane after 15 minutes?

$h = 9000 - 300t$
$h = 9000 - 300(15)$
$h = 9000 - 4500 = 4500m$

c) How long after the plane begins its decent does the plane land?

- when the plane lands the height is 0, so find the time when h = 0.

$h = 9000 - 300t$
$0 = 9000 - 300t$
$0 = 9000 - 300 \cdot \underline{30}$
$t = 30$ mins

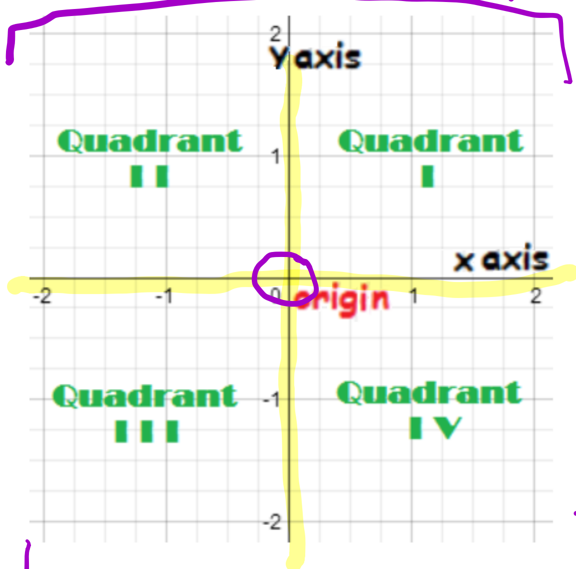
Section 4.2 – Linear Relations

The graph we use to plot points is called the **Cartesian Coordinate System**. It consists of:

- x – axis, which goes horizontal (across)
- y – axis, which goes vertical (up/down)
- Origin, the point where the x and y axes meet. It's coordinates are (0, 0).
- 4 quadrants

+ y

negative x -



+ x

A point consists of two coordinates: (x, y)

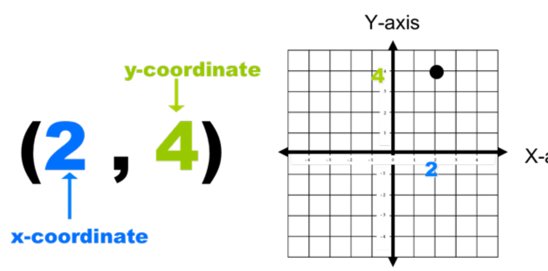
- y

- The first number represents x; it's the distance to move in the horizontal direction.

  negative # - move left   positive # - move right

- The second number represents y; it's the distance to move in the vertical direction.

  negative # - move down   positive # - move up



- x always goes first!

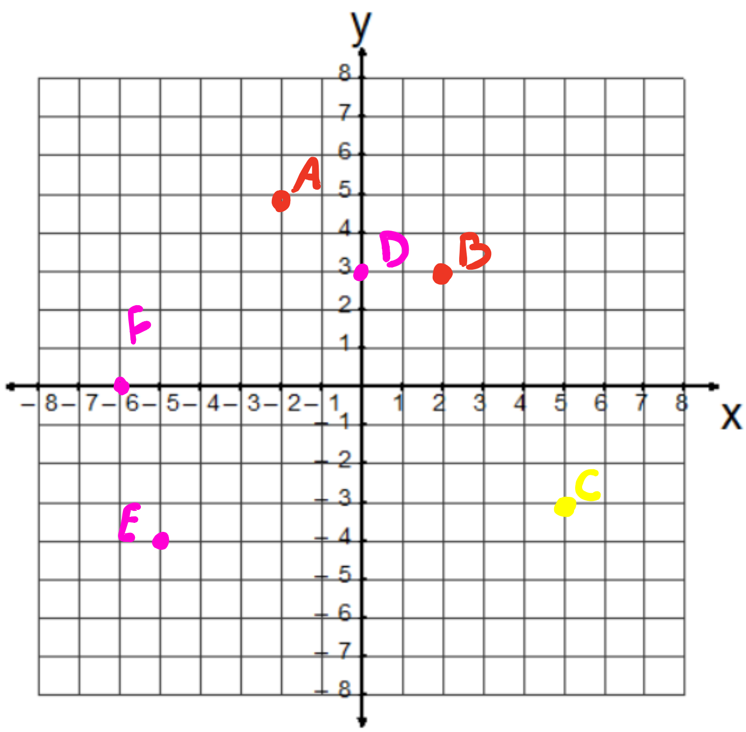
- Remember each point consists of two numbers which are directions to get you to one point.

Example:

Plot the following points on the coordinate grid below.

$A(-2,5)$ $C(5,-3)$ $E(-5,-4)$

$B(2,3)$ $D(0,3)$ $F(-6,0)$

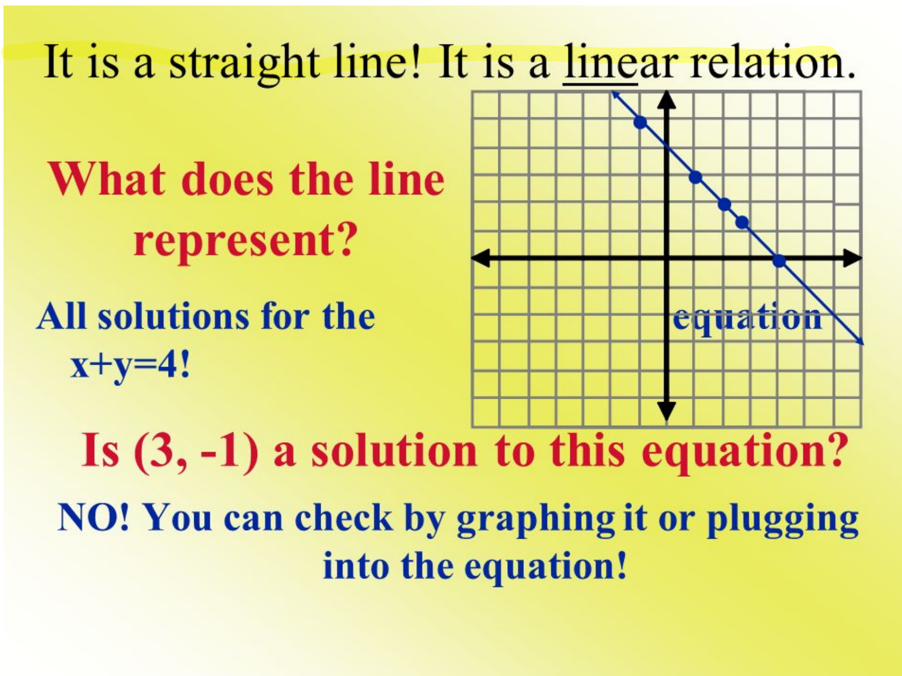


When creating a graph for a particular problem, for example hours worked and amount earned or distance travelled over time or figure number and number of squares, etc. we need to remember these important things about graphing:

- Increase by the same amount on each axis. For example, the x – axis could increase by 1 and the y – axis could increase by 10, but be consistent on each axis.
- Label each axis
- Title the graph
- Independent variable goes on the x – axis, dependent variable goes on the y – axis.

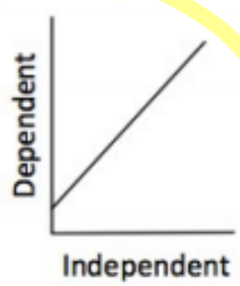
Linear Relation

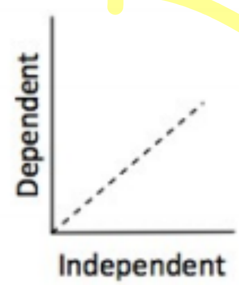
- When the relationship between the independent and dependent variables can be represented in a straight line graph.
- A constant change in one variable produces a constant change in the other variable.

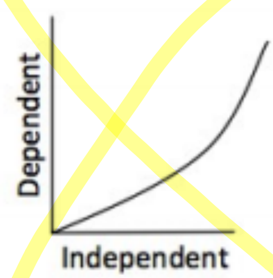


Example:

Which graph(s) represents a linear relation?







Example:

Refer to each table below.

a) Does it represent a linear relation?
b) If the relation is linear, describe it and write an equation.
c) If the relation is not linear, how do you know?

i)

ind dep

x	y
1	4
2	6
3	8
4	10
5	12

$4 = 2(1)+2$
$4 = 4$ ✓

a) constant change in the dependant ∴ linear

b) ind. variable increases by 1, dep. variable increases by 2

$y = 2x + 2$

ii)

h	C
1	1
2	4
3	7
4	10
5	13

$1 = 3(1) - 2$
$1 = 1$ ✓

a) linear

b) ind. ↑1, dep. ↑3

$C = 3h - 2$

iii)

x	y
1	9
2	8
3	7
4	6
5	5

a) linear

b) ind ↑1, dep ↓1

c) $y = 10 - x$ or $y = -x + 10$

iv)

x	y
1	2
2	5
3	10
4	17
5	26

3
5
7
9

a) non-linear

b) not constant

Example:

Complete the following tables, using the equations provided.

a) $y = 2x + 1$

x	y
1	3
2	5
3	7
4	9
5	11

b) $y = 10 - x$

x	y
7	3
5	5
4	6
2	8
0	10

graphing linear equations

graph $y = 2x + 1$

construct a table of values

x	-2	-1	0	1	2
$y = 2x + 1$	-3	-1	1	3	5

Now, let's write ordered pairs, and graph.

Example:

A scuba diver goes under water. The deeper he goes, the more water pressure he feels. Refer to the table to see the relationship between the depth and water pressure.

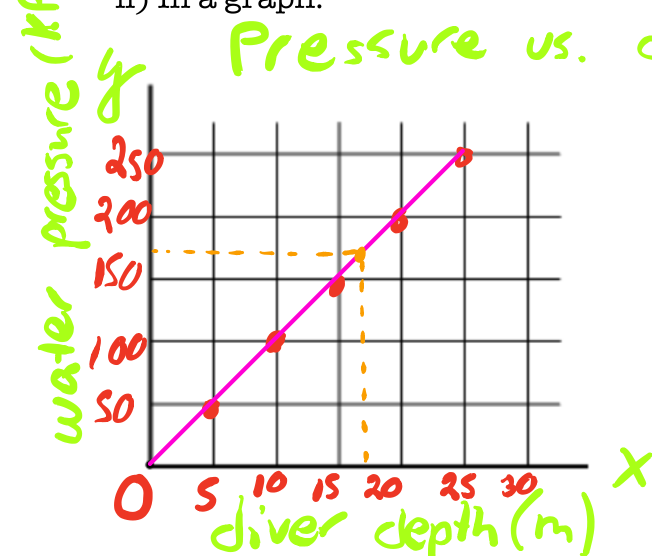
Diver's Depth (m)	Water Pressure (kPa)
0	0
5	50
10	100
15	150
20	200

a) Describe the relationship between the diver's depth and water pressure....

i) In words:

for every 5m the diver goes down, the pressure increases by 50KPa

ii) In a graph:



Does it make sense to connect the points?

Continuous Data

- We can find values between the plotted points
- In the graph, the points are connected

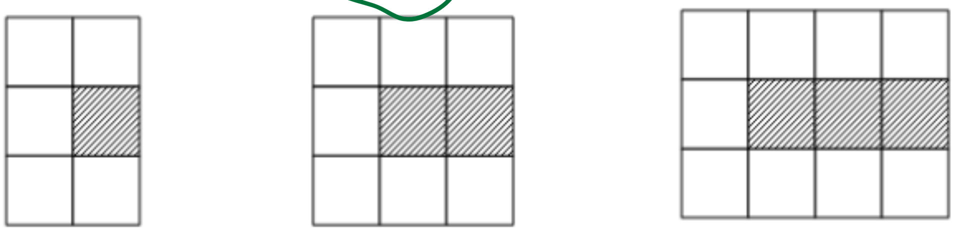
iii) In an equation:

$P = 10d$

$50 = 10(5)$

Example:

Suppose the following pattern is continued.



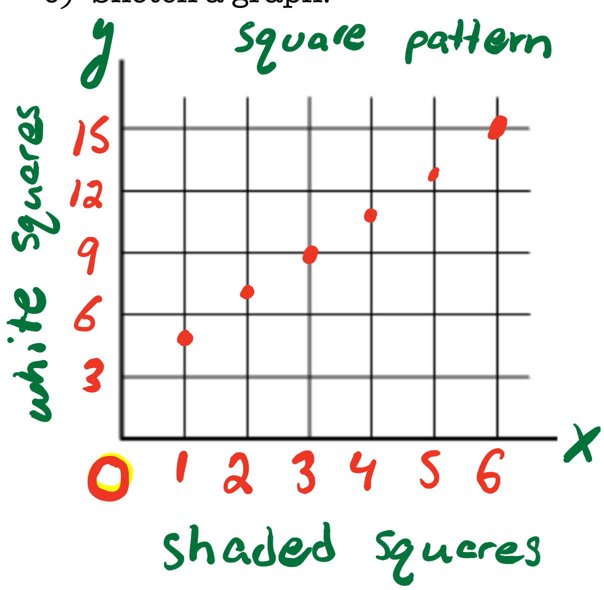
a) Complete the following table for the first 6 diagrams in this pattern.

Number of Shaded Squares	1	2	3	4	5	6
Number of White Squares	5	7	9	11	13	15

b) Write an equation.

$$W = 2s + 3$$

c) Sketch a graph.



Does it make sense to connect the points?

Discrete Data

- We cannot find values between the plotted points
- In the graph, the points are not connected

Finding Missing Value From An Equation

Example:

For each equation, find the missing value.

a) Using the linear relation $y = 2x + 5$

i) What is the value of y if $x = 3$?

$y = 2(3) + 5 = 11$

ii) What is the value of x if $y = 25$?

$25 = 2(x) + 5 \rightarrow \frac{20}{2} = \frac{2x}{2} \rightarrow 10 = x$

b) Using the linear relation $y = 3x - 4$

i) What is the value of y if $x = 6$?

$y = 3(6) - 4 = 14$

ii) What is the value of x if $y = 23$?

$23 = 3(x) - 4 \rightarrow 27 = 3(x) \rightarrow x = 9$

c) Using the linear relation $y = 1 - 2x$

i) What is the value of y if $x = -2$?

$y = 1 - (-2)(2) = 5$

ii) What is the value of x if $y = 9$?

$9 = 1 - 2x$

$8 = -2x$

$-4 = x$

Finding Missing Value From a Table

Example:

For each table, find the missing values.

a)

x	y
1	4
2	13
3	22
4	31
5	40

9
9
9
9

b)

x	y
0	2
2	8
4	14
6	20
8	26

6
6

c)

x	y
3	-9
4	-8
5	-7
6	-6
7	-5

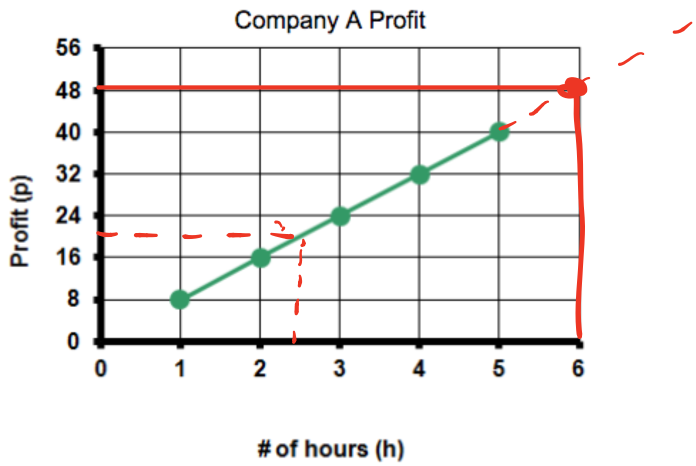
+1

Finding Missing Value From a Graph

Example:

For each graph, find the missing value.

a) What is the profit for working 2.5 hours?



Interpolating

└→ estimate a value between two plotted points

\$20

b) What is the profit for working 6 hours?

Extrapolating

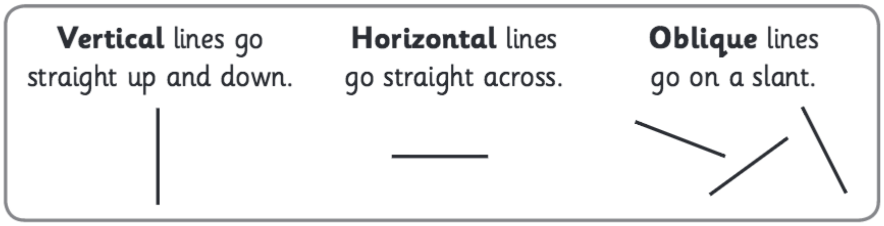
└→ estimate a value that lies beyond the plotted points.

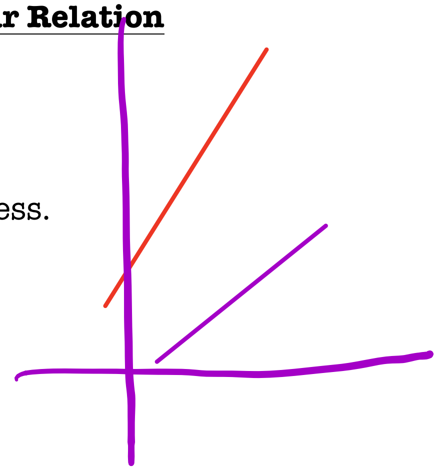
\$48

Section 4.3 – Another Form of the Equation for a Linear Relation

Oblique Lines

Oblique lines are slanted or diagonal lines.
They can slant in either direction and can have different steepness.
They will always contain two variables.





Example:

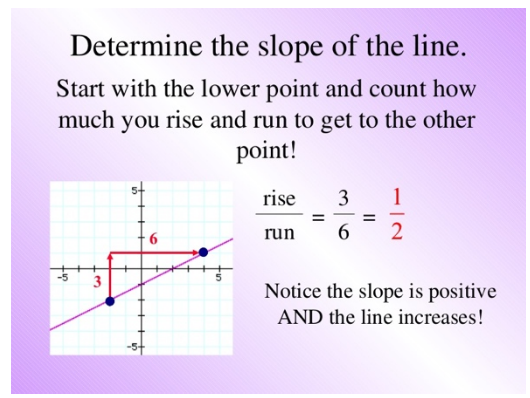
For each of the following:

(i) Complete each table of values.

(ii) Graph each line on the coordinate grid provided.

(iii) Describe each relation.

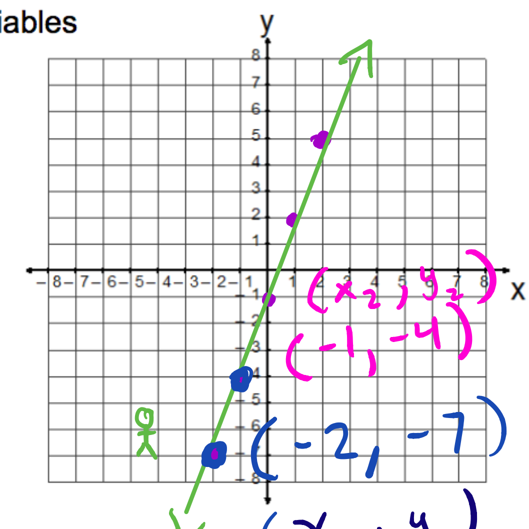
(iv) Identify the slope.



$y = 3(-1) - 1$
$-3 - 1$

a) $y = 3x - 1$ ← two variables

x	y
-2	-7
-1	-4
0	-1
1	2
2	5



$y = 3x - 1$
$y = 3(-2) - 1$

$m =$ slope $= \frac{rise}{run} = \frac{\Delta y}{\Delta x}$

$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-4) - (-7)}{(-1) - (-2)} = \frac{-4 + 7}{-1 + 2} = \frac{3}{1} = 3$

Which direction is the line slanting?

Up = increase(+)

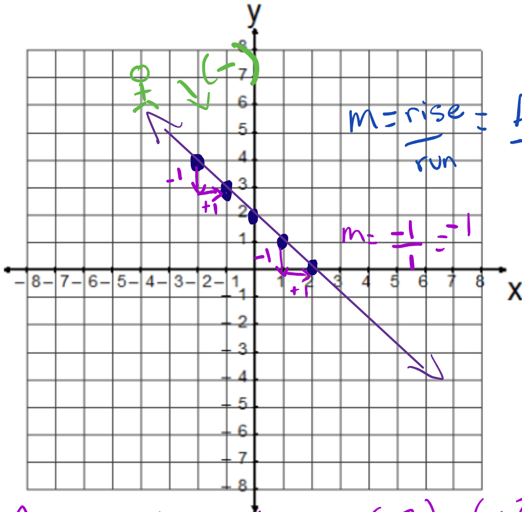
b) $y = -x + 2$

$y = mx + b$

x	y
-2	4
-1	3
0	2
1	1
2	0

>-1

$y = -(-2) + 2$
$= +2 + 2 = 4$



$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{-1}{1} = -1$

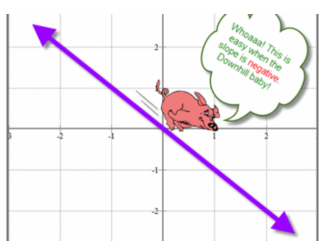
slope = $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(2)-(-2)}{(0)-(4)} = \frac{2+2}{-4} = \frac{4}{-4} = -$

Which direction is the line slanting?

down

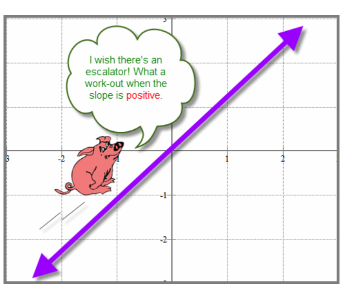
Notes:

- When the independent variable, x, increases by 1 and the dependent variable, y, **decreases**, the line will **slant down**.



This is represented in the equation $y = -2x + 1$, with the **negative** number in front of x.

- When the independent variable, x, increases by 1 and the dependent variable, y, **increases**, the line will **slant up**.

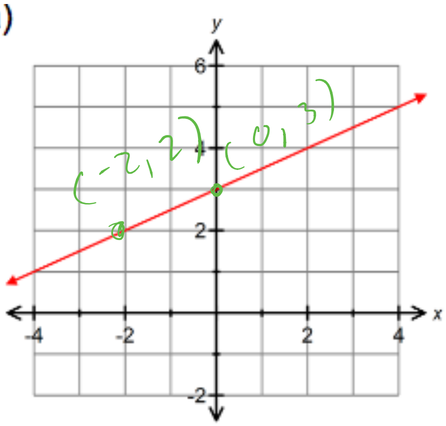


This is represented in the equation $y = 2x - 1$, with the **positive** number in front of x.

Example:

Calculate the slope of each line.

(a)

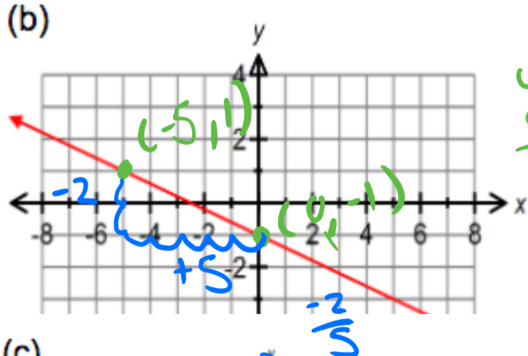


$slope = \frac{rise}{run} = \frac{1}{2}$

$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(3)-(2)}{(0)-(-2)}$

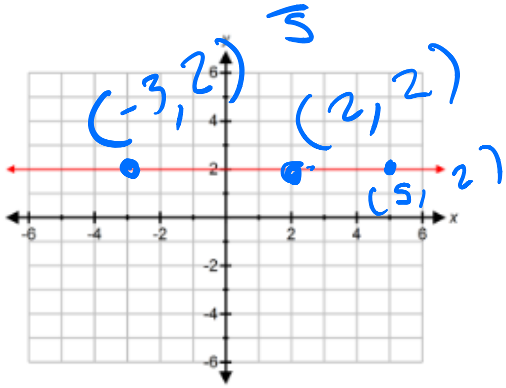
$= \frac{1}{+2} = \frac{1}{2}$

(b)



$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(1)-(-1)}{(-5)-(0)} = \frac{2}{-5} = \frac{-2}{5}$

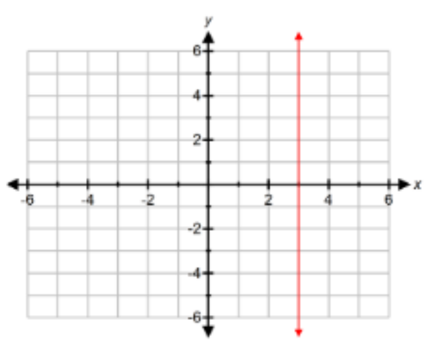
(c)



$y = 2$

$slope = \frac{0}{12} = 0$

(d)



$x = 3$

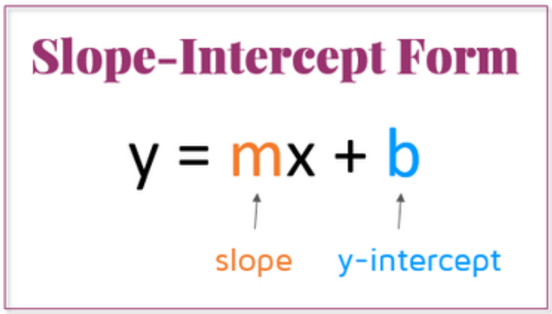
$slope = \frac{12}{0}$

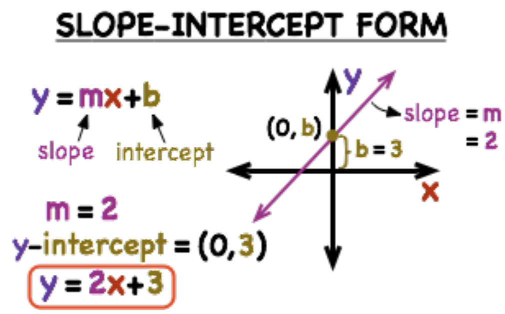
undefined

Equation of a Line

Lines can be written in the form: $y = mx + b$

This is called **Slope-Intercept Form** for the equation of a line.





the point where the line crosses the y-axis.

Example:

What is the slope and y-intercept of each line?

a) $y = 2x + 5$ slope = 2 y-int = 5 (0, 5)

b) $y = -\frac{1}{2}x + 4$ slope: $-\frac{1}{2}$, (0, 4)

c) $y = \frac{2}{3}x + 11$ slope = $\frac{2}{3}$, (0, 11)

d) $y = 4x - 3$ y-intercept (b)

slope (m)

e) $y = 3x - \frac{1}{4}$ y-int.

slope

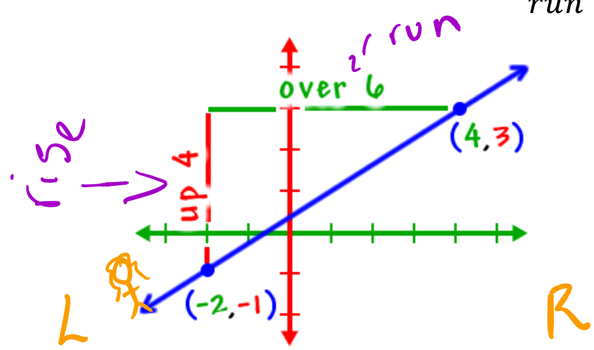
We can find the slope of a line several ways.

- **From a graph**

  Choose any two points and calculate $\frac{rise}{run}$.

  ← y-points

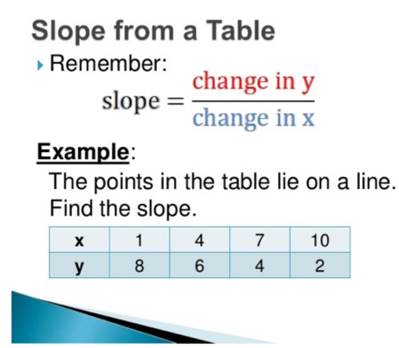
  ← x-point



$m = \frac{4}{6} = \frac{2}{3}$

- **From a table of values**

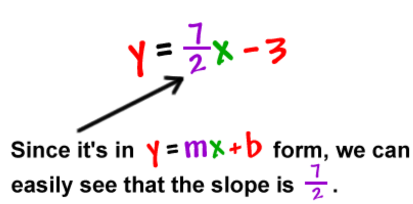
  The slope is found by calculating $\frac{change\ in\ dependent\ variable\ (y)}{change\ in\ independent\ variable\ (x)} = \frac{\Delta y}{\Delta x}$.



x | y

- **From an Equation**

  The slope is the number multiplied by x, when written in slope-intercept form.



Example:

What is the equation of each line, when given slope and y-intercept?

a) slope = $\frac{2}{3}$, y – intercept = 7 $\rightarrow y = \frac{2}{3}x + 7$

b) slope = – 4 , y-intercept = – 2 $\rightarrow y = -4x - 2$

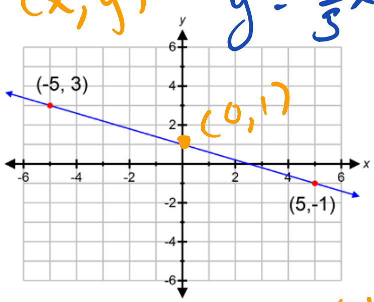
c) slope = 0 , y-intercept = 3 $\rightarrow y = 0x + 3 \rightarrow y = 3$

d) slope = $-\frac{1}{2}$, y-intercept = 0 $\rightarrow y = -\frac{1}{2}x + 0 \rightarrow y = -\frac{1}{2}x$

Example:

What is the slope, y-intercept and equation of each line graphed below?

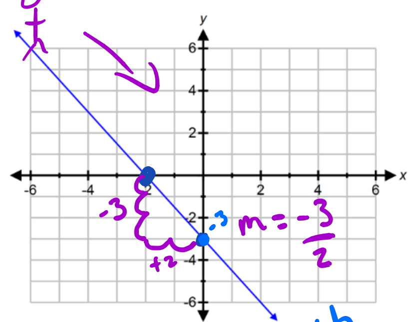
a) (x, y) $y = -\frac{2}{3}x + 1$



$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1) - (3)}{(5) - (-5)}$$

$$= \frac{-1 - 3}{5 + 5} = \frac{-4}{10} = -\frac{2}{5}$$

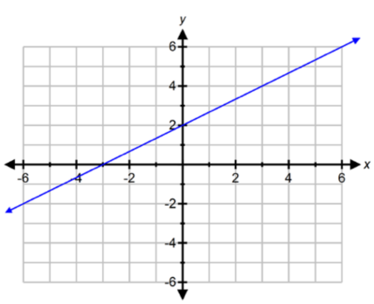
b) 0 + x



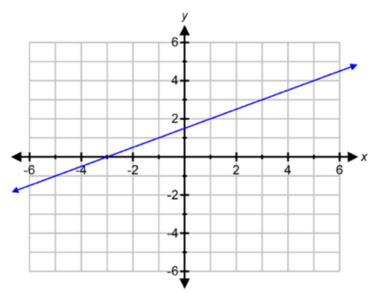
$y = mx + b$

$y = -\frac{3}{2} - 3$

c)



d)



The equation of a line is not always written in slope-intercept form. We can also represent lines in other forms.

Another way to write the equation of a line is when the equation has both variables on the same side.

For example, $2y - 4x = 8$ is also a way of writing a linear relation.

To graph this relation, first we would need to "solve for y" (rewrite it in "slope-intercept form"). Then create a table of values and plot the corresponding points on a coordinate grid.

Slope-int. form: $y = mx + b$
Standard form: $ax + by = c$
Point-slope: $y - y_1 = m(x - x_1)$

- Solve for y

$$2y - 4x = 8$$
$$2y - 4x + 4x = 8 + 4x$$
$$2y = 4x + 8$$
$$\frac{2y}{2} = \frac{4x}{2} + \frac{8}{2}$$
$$y = 2x + 4$$

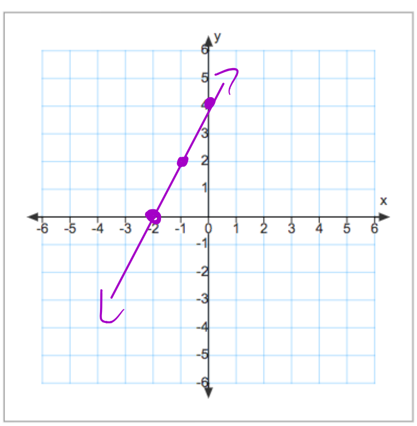
- Create a table of values

x	*y*
-2	0
-1	2
0	4
1	6
2	8

+2

$y = 2x+4$
$y = 2(-2)+4 = 0$
$y = 2(-1)+4 = 2$

- Draw a graph



Example:

For each equation below:

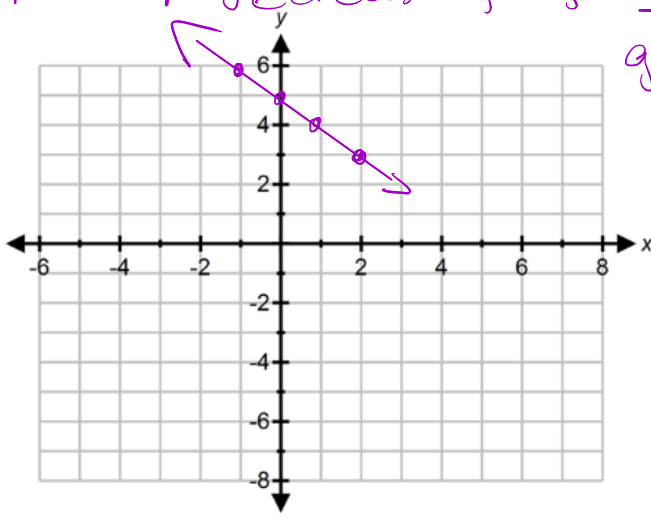
i) Complete each table of values.
ii) Describe each relation.
iii) Which way will the graph slant?
iv) Graph the relation.

a) $x + y = 5$

↳ standard form

The independent (x) is increasing by one
The dependent (y) is decreasing by one ∴ graph slants down

x	y
-2	7
-1	6
0	5
1	4
2	3



$y = -x + 5$

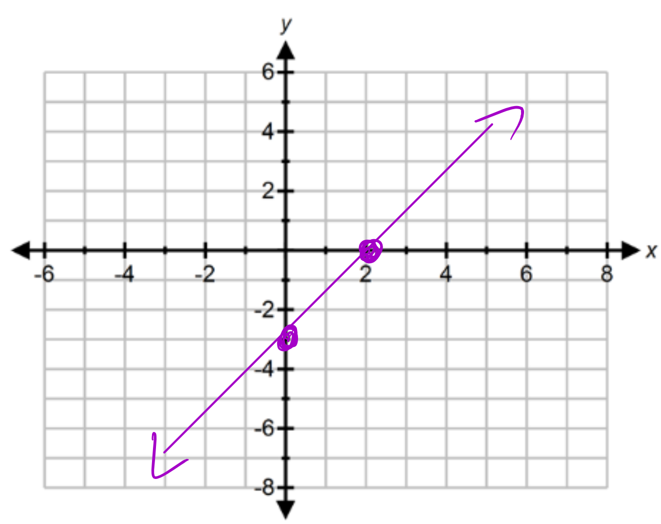
$y = -(-2) + 5$

$y = -(-1) + 5$

b) $3x - 2y = 6$

$y = \frac{3}{2}x - 3$

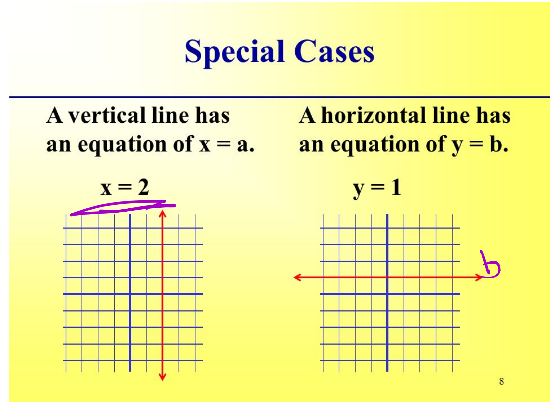
x	y
-2	
-1	
0	-3
1	
2	0



$y = \frac{3}{2}(2) - 3$

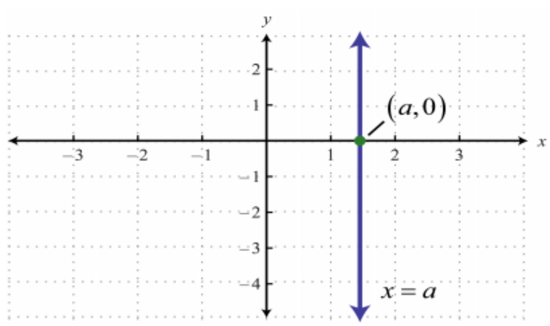
$y = 3 - 3 = 0$

Special Cases for an Equation of a Line

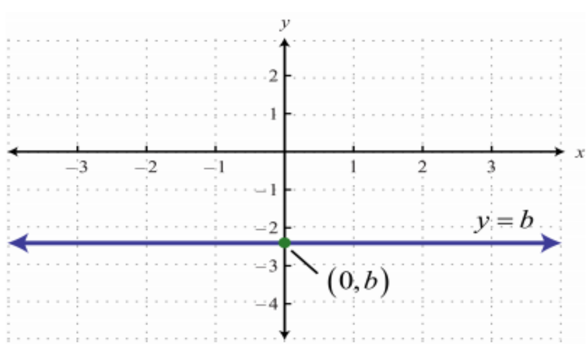


Sometimes only one variable will appear in the equation. When this happens we would have either $\boldsymbol{x}$ = "***some number***" or $\boldsymbol{y}$ = "***some number***".

When the equation is $\boldsymbol{x}$ = "***some number***", the graph is a **vertical line** and is perpendicular to the x-axis. Every point on the graph has an x-coordinate of "*that number*".



When the equation is $\boldsymbol{y}$ = "***some number***", the graph is a **horizontal line** and is perpendicular to the y-axis. Every point on the graph has an y-coordinate of "*that number*".



homework

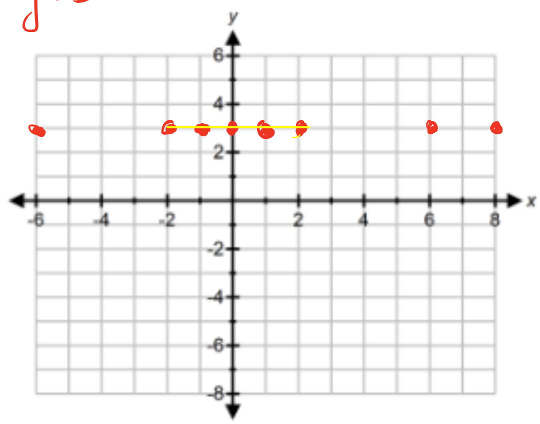
Example:

Graph each line and then describe it.

a)

x	y
-2	3
-1	3
0	3
1	3
2	3

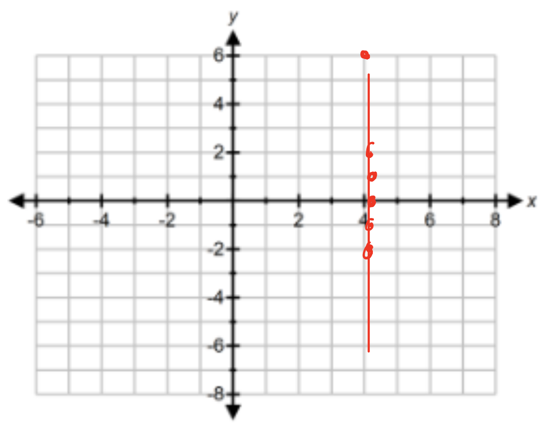
y = 3



- straight horizontal line
- parallel to x
- perp. to y

b)

x	y
4	-2
4	-1
4	0
4	1
4	2

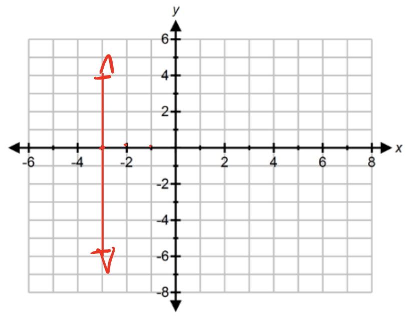


homework

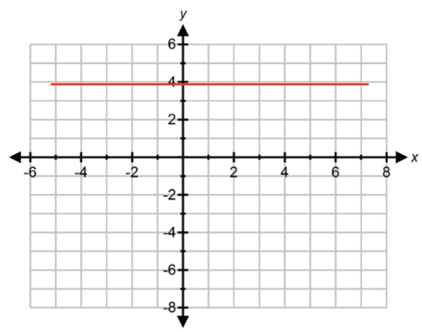
Example:

Graph each of the following equations. Then describe the graph.

a) $x = -3$

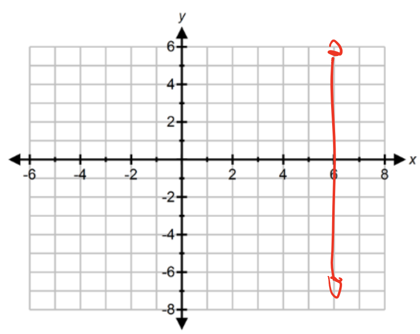


b) $y = 4$



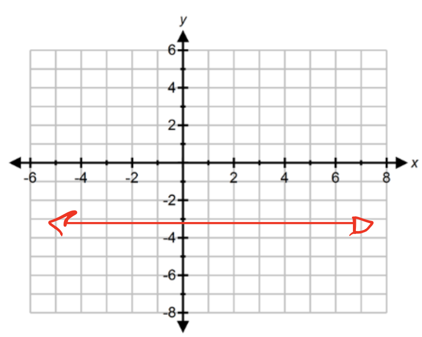
c) $x - 4 = 2$

$x = 6$



d) $y + 3 = 0$

$y = -3$



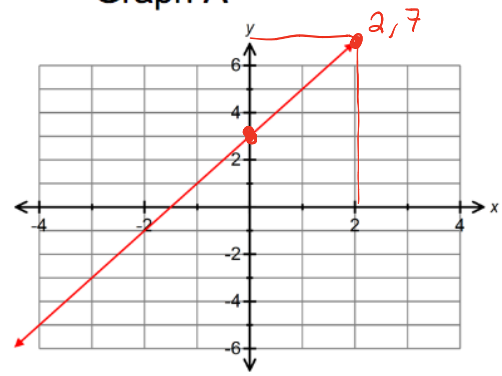
Section 4.4 – Matching Equations & Graphs

The 3 graphs below have the equations:

$y = 2x - 3$, $y = -2x$ and $y = 2x + 3$

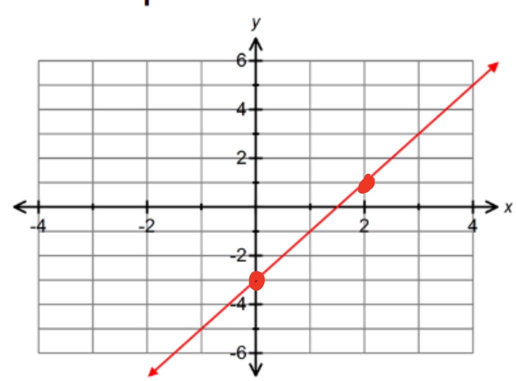
Equation # 1	**y = 2x – 3**	B
Substitute x = -2	y = 2(-2) – 3 Y = -7	(-2, -7)
Substitute x = 0	y = 2(0) – 3 Y = -3	(0, -3)
Substitute x = 2	y = 2(2) – 3 Y = 1	(2, 1)
Which Graph has these 3 points?		

Graph A



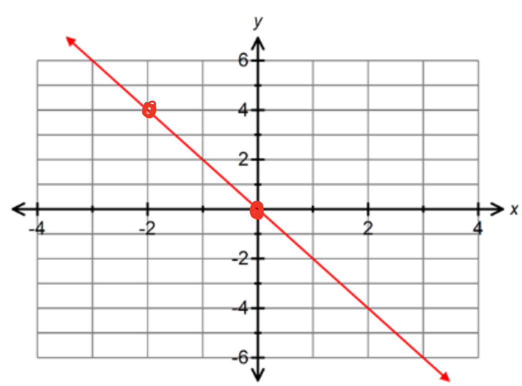
Equation # 2	**y = -2x**	C
Substitute x = -2	y = -2 (-2) Y = 4	(-2, 4)
Substitute x = 0	y = -2(0) Y = 0	(0, 0)
Substitute x = 2	y = -2(2) Y = -4	(2, - 4)
Which Graph has these 3 points?		

Graph B



Equation # 3	**y = 2x + 3**	A
Substitute x = -2	y = 2(-2) + 3 Y = -1	(-2, -1)
Substitute x = 0	y = 2(0) + 3 Y = 3	(0, 3)
Substitute x = 2	y = 2(2) + 3 Y = 7	(2, 7)
Which Graph has these 3 points?		

Graph C



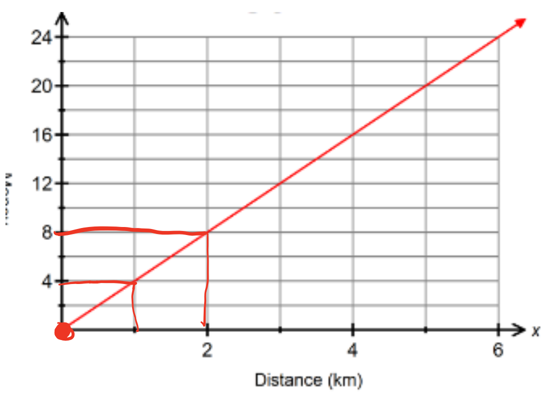
Example:

Ben, Michael and Sam participate in a 5 km walk for charity. Each student has a different plan to raise money from their sponsors. The following graphs show how the amount of money a sponsor owes is related to the distance walked.

Match each equation with its graph.

$\cancel{M = 4d}$ $\cancel{M = 2d + 3}$ $\cancel{M = d + 5}$

Sam



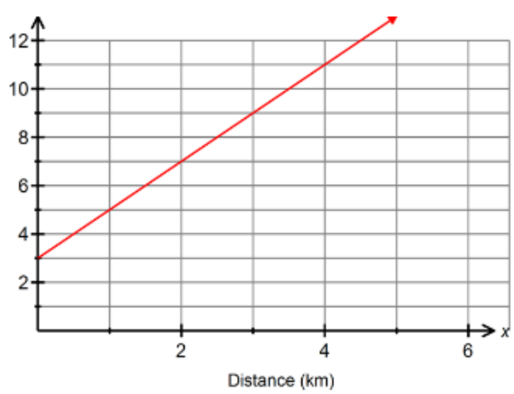
Describe how Sam is collecting money from his sponsors.

Sam collects $4 for every Km walked

d	m
0	0
1	4
2	8
3	12
4	16

Equation: $M = 4d$

Michael



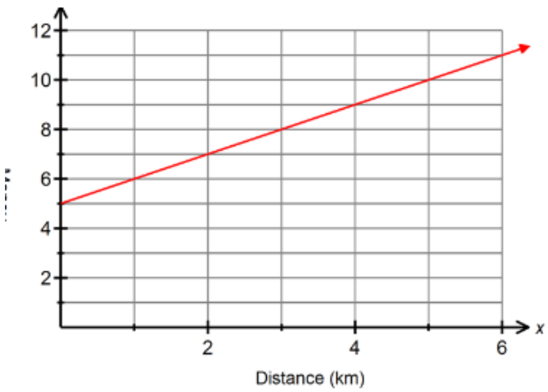
Describe how Michael is collecting money from his sponsors.

d	m
0	3
1	5
2	7
3	9
4	11

Equation: $M = 2d + 3$

~> Michael was given $3 at the start, and $2 for every Km walked

Ben



Describe how Ben is collecting money from his sponsors.

d	m
0	5
1	6
2	7
3	8
4	9

Equation: $M = d + 5$

\$5 to start

\$1 per km

Example:

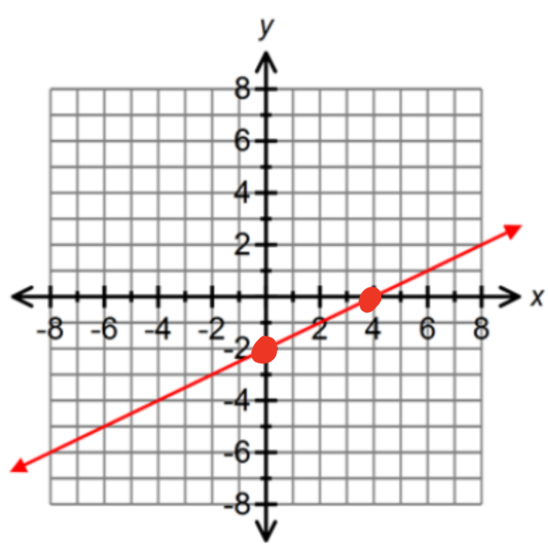
Why does the equation and graph match?

$x - 2y = 4$

$-2y = 4 - x$

$y = \frac{4 - x}{-2}$

$y = \frac{x - 4}{2}$

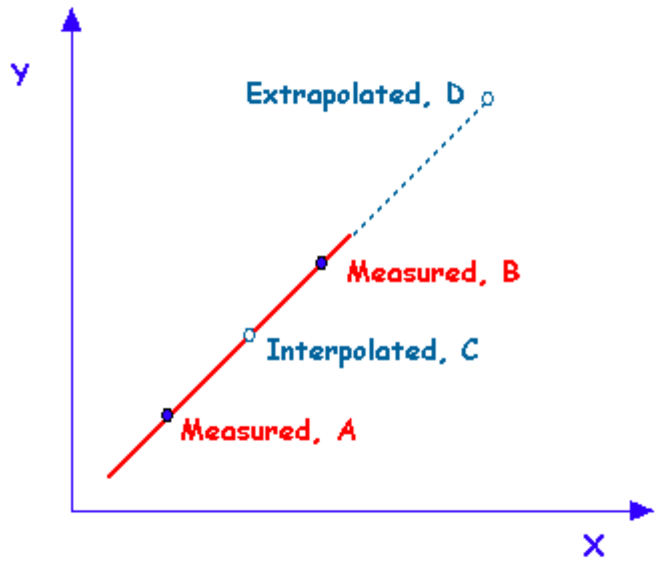


$y = \frac{0 - 4}{2} = \frac{-4}{2} = -2$

$y = \frac{4 - 4}{2} = 0$

Section 4.5 – Using Graphs to Estimate Values

We can estimate values on a graph using **interpolation** and **extrapolation**.

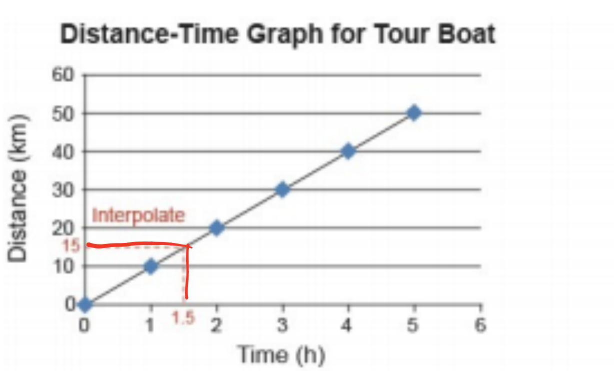


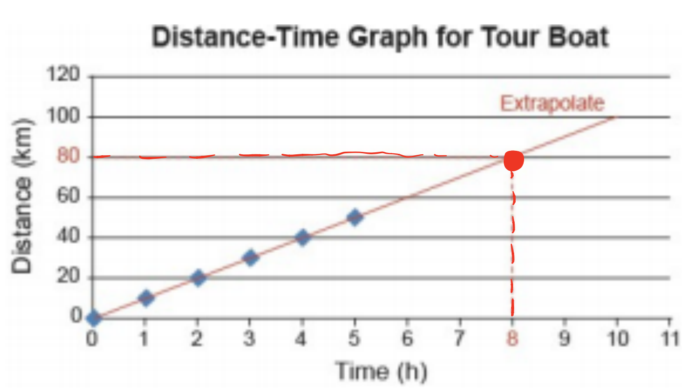
Interpolation is when we estimate values between 2 given data points on a graph of a linear relation.

Extrapolation is when we extend a graph of a linear relation to estimate values that extend beyond the graph.

How do we do this?

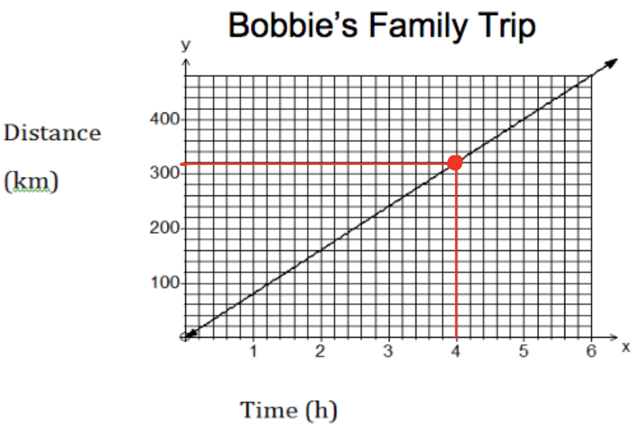
To interpolate/extrapolate, we need to locate the point in question, draw a horizontal/vertical line from that point, then draw a vertical/horizontal line from the graph to the horizontal/vertical axis.





Example:

Consider the graph,



a) How long did it take Bobbie's family to travel 320 km?

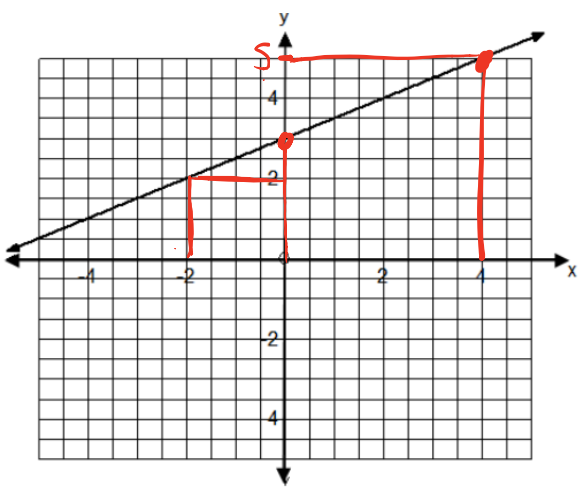
4hrs

b) How long will it take Bobbie's family to travel 640 km?

$4 \times 2 = 8$hrs

Example:

Use the graph to answer the following questions below.



a) Determine the value of y when $x = -2$. $x=-2,\ y=2$

b) Determine the value of x when $y = 5$. $x=4$

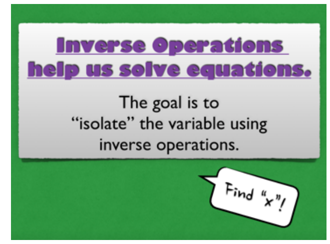
c) Determine the value of x when $y = 3$.

$x=0$

4.3 practice

p301 practice

Section 6.1 – Solving Equations Using Inverse Operations



Inverse operations "undo" or reverse each other's results.

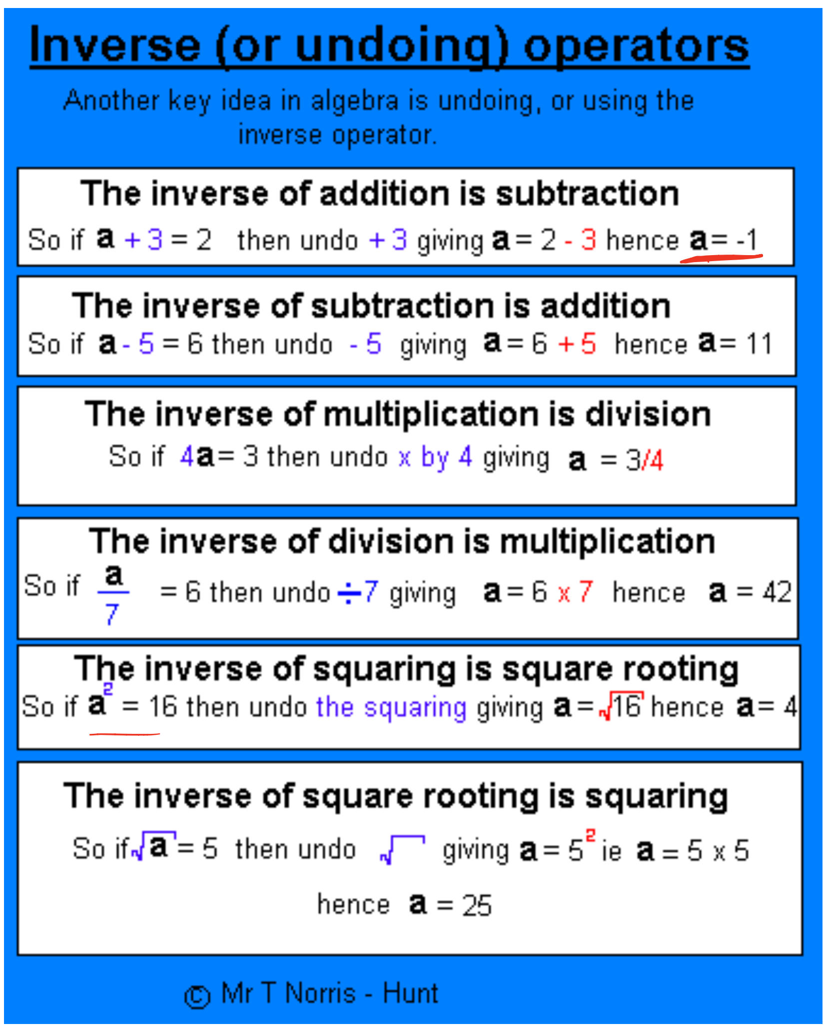
Inverse operations take us back to where we started!

Inverses "Undo" Each Other

Addition	cancels	Subtraction
Multiplication	cancels	Division

For example, $2 + 3 = 5$ and $5 - 3 = 2$

$2 \times 3 = 6$ and $\frac{6}{3} = 2$



We can use inverse operations to solve many types of equations.

To do this, we:

1. Determine the operations that were applied to the variable to **build** the equation

2. Then use the **inverse operation** to isolate the variable (get x by itself) by "undoing" the operation.

Keeping in mind that whatever we do to one side of the equation, we do to the other side to keep the equation **"balanced"**.

How to Isolate a Variable

- Use Inverse Operations
- Addition $\longleftrightarrow$ Subtraction
- Multiplication $\longleftrightarrow$ Division
- Do the same thing to both sides of the equation
- Do Add/Subtract before Multiply/Divide

For example, to solve $x + 2.4 = 6.5$

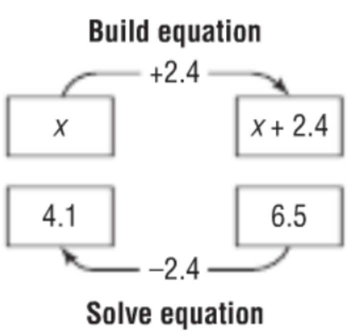
Step 1: Start with x

Step 2: Identify the operation applied to produce $x + 2.4$

add 2.4

Step 3: Apply the inverse operation to isolate x.

subtract 2.4



Algebraically,

$$x + 2.4 = 6.5$$

$$\begin{array}{r} x + 2.4 = 6.5 \\ -2.4 \quad -2.4 \\ \hline x = 4.1 \end{array}$$

We can verify our solution:

- put your answer back into the equation to show that it works.

Example:

Solve each of the following using inverse operations and algebraically.

Grade 7

a) $x - 4.3 = -5.6$ Operation: Subtraction

$x - 4.3 = -5.6$

Inverse Operation	Algebraically
$+4.3$	$x - 4.3 \; +4.3 = -5.6 \; +4.3$
	$x = -1.3$

b) Three times a number is -3.6 Operation: multiplication

n

$3 \times n = -3.6$ Inverse: division

Inverse Operation	Algebraically
$3n = -3.6$	$\frac{3n}{3} = \frac{-3.6}{3}$
	$n = -1.2$
	$\boxed{n = -1.2}$

c) A number divided by 4 is 1.5 Operation: division

$\frac{x}{4} = 1.5$ Inverse: multiplication

Inverse Operation	Algebraically
$\cancel{4}$	$\cdot \frac{x}{\cancel{4}} = 1.5 \cdot 4$
	$x = 6$

Solving Two Step Equations

* * When more than one operation acts on a variable in an algebraic equation, apply the **reverse** of the order of operations to reverse the operations. Here is the order in which you should reverse operations:

1. Reverse addition and subtraction (by subtracting and adding) outside brackets.

2. Reverse multiplication and division (by dividing and multiplying) outside brackets.

3. Remove (outermost) brackets, and reverse the operations in order according to these three steps.

NOTE: To "undo" a sequence of operations, we perform the inverse operations in the **reverse** order.

In other words, we have to "undo" the last step first!

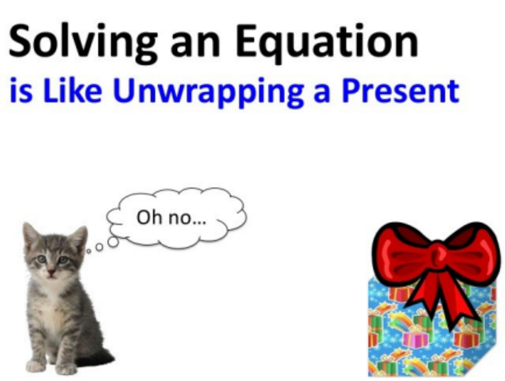
Example:

Wrapping A Present

(1) Put Present in box. (2) Wrap Box. (3) Put on Bow.

Unwrapping A Present

(3) Take Present out of Box. (2) Unwrap Box. (1) Take off Bow.



$5x + 9 = 44$

let $x = 7$

$5(7) + 9 \stackrel{?}{=} 44$

$35 + 9 \stackrel{?}{=} 44$

$44 = 44$ ✓

Example:

Solve each of the following equations.

a) $5x + 9 = 44$
- Reverse addition: Subtract 9
- Reverse multiplication: Divide 5
- Verify your answer:

1. $5x + 9 = 44$
2. $5x = 44 - 9$
3. $\frac{5x}{5} = \frac{35}{5}$
4. $\boxed{x = 7}$

b) $4.5d - 3.2 = -18.5$
1. • Reverse subtraction: Add 3.2
2. • Reverse multiplication: divide 4.5
- Verify your answer:

$4.5(-3.4) - 3.2 \stackrel{?}{=} -18.5$

$-18.5 = -18.5$ ✓

$4.5d - 3.2 = -18.5$

$4.5d = -18.5 + 3.2$

$\frac{4.5d}{4.5} = \frac{-15.3}{4.5}$

$\boxed{d = -3.4}$

$45 \overline{)153}$

4.5×3.4 } 2 decimal places

180

1350

-15.30

c) $\frac{x}{4} + 3 = 7.2$
- Reverse addition:
- Reverse division:
- Verify your answer:

d) $3\left(\frac{y}{4} - 1\right) = 15$
- Reverse multiplication:
- Reverse subtraction:
- Reverse division:
- Verify your answer:

$\frac{\cancel{3}\left(\frac{y}{4} - 1\right)}{\cancel{3}} = \frac{15}{3}$

$\frac{y}{4} - 1 = 5$

$\frac{y}{4} = 5 + 1$

$4 \times \frac{y}{4} = 6 \times 4$

$\boxed{y = 24}$

$3\left(\frac{y}{4} - 1\right) = 15$

$3\left(\frac{24}{4} - 1\right) \stackrel{?}{=} 15$

$3(6 - 1) \stackrel{?}{=} 15$

$3(5) \stackrel{?}{=} 15$
$15 = 15$ ✓

e) $\mathbf{4(3(z-11)+6)=48}$
- Reverse multiplication:
- Reverse addition:
- Reverse multiplication:
- Reverse subtraction:
- Verify your answer:

$4(3(13-11)+6 \stackrel{?}{=} 48$
$4(3(2))+6) \stackrel{?}{=} 48$
$4(12) \stackrel{?}{=} 48$
$48 = 48$ ✓

$\frac{\cancel{4}(3(z-11)+6)}{\cancel{4}} = \frac{48}{4}$
$3(z-11)+6 = 12$
$3(z-11) = 12-6$
$\frac{3(z-11)}{3} = \frac{6}{3}$
$z-11 = 2$
$z = 2+11$
$z = 13$

NOTE: Sometimes, the equation will not start out simplified. If this is the case, simplify the equation ***before*** reversing the operations.

Example:

Solve for x: $\mathbf{6x - 5 - 2x + 3 - 2 = 4}$

- Simplify the equation first (combine like terms):
- Reverse subtraction:
- Reverse multiplication:
- Verify your answer:

$x = 2$

Example:

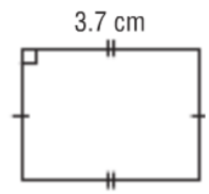
Seven percent of a number is 56.7.

a) Write, then solve an equation to determine the number.

b) Check the solution.

Example:

A rectangle has a length 3.7 cm and a perimeter 13.2 cm.



a) Write an equation that can be used to determine the width of the rectangle.

b) Solve the equation.

c) Verify the solution.

Solving Equations Involving the Distributive Property

Example:

Solve each of the following equations using the distributive property.

$3.7 \times 2 = 7.4$

a) $2(3.7 + x) = 13.2$

$7.4 + 2x = 13.2 - 7.4$

$\frac{2x}{2} = \frac{5.8}{2}$

$x = 2.9$

b) $6 = 1.5(x - 6)$

$\frac{6}{1.5} = \frac{1.5(x-6)}{1.5}$

$4 = x - 6$

$4 + 6 = x$

$x = 10$

$10 = x$

c) $3(x - 5) = \frac{2}{3}$

$\frac{3(x-5)}{3} = \frac{2}{3}$

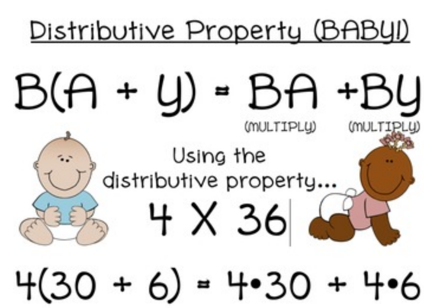
$x - 5 = \frac{2}{3}$

$x = \frac{2}{3} + 5$

$x = \frac{2}{3} + \frac{5}{1}\left(\frac{3}{3}\right)$

$x = \frac{2}{3} + \frac{15}{3} = \frac{17}{3}$

$x = 5.\overline{6}$

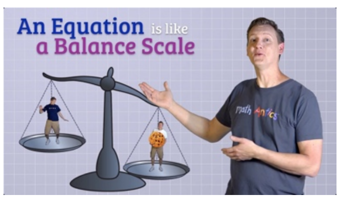


Section 6.2 – Solving Equations Using Balance Strategies

To solve an equation, we need to isolate the variable, which means get it by itself.

In the last section we used inverse operations. That method only works, however, when the variable occurs only once in the equation.

Another way to isolate the variable is to use a **balance strategy**.

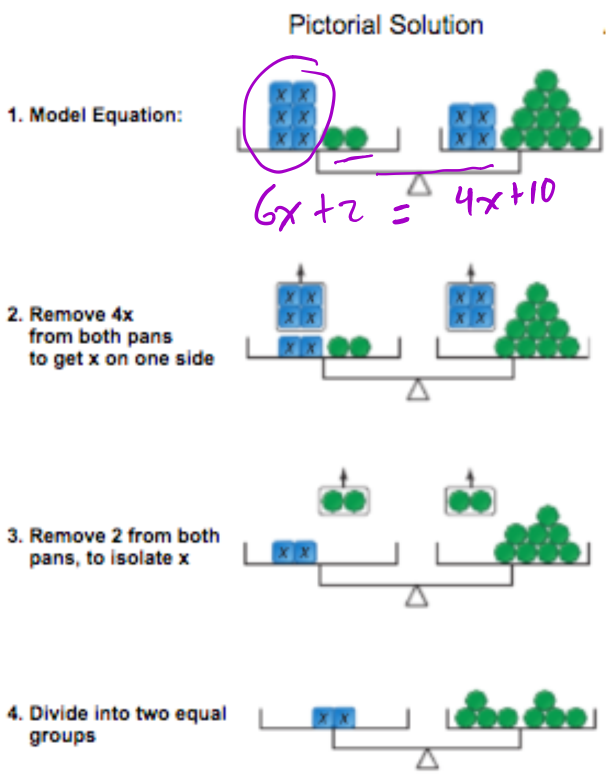


To keep the scale "balanced", **whatever we do to one side of the "scale"/equation, we must do to the other side**.

To solve, we need to get the variable on one side of the equal sign and the constant term on the other.

Modelling Equations with Variable on Both Sides

For example, let's look at the following equation: $6x + 2 = 10 + 4x$



you do

Algebraic Solution

$6x + 2 = 10 + 4x$

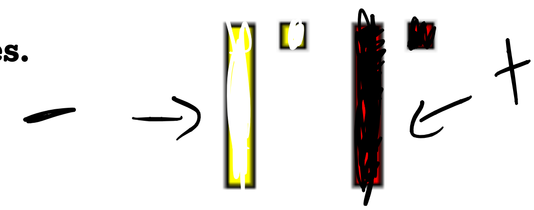
$6x - 4x = 10 - 2$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

We cannot use a balance scale model when any term in the equation is negative.

Another strategy is to use **algebra tiles.**



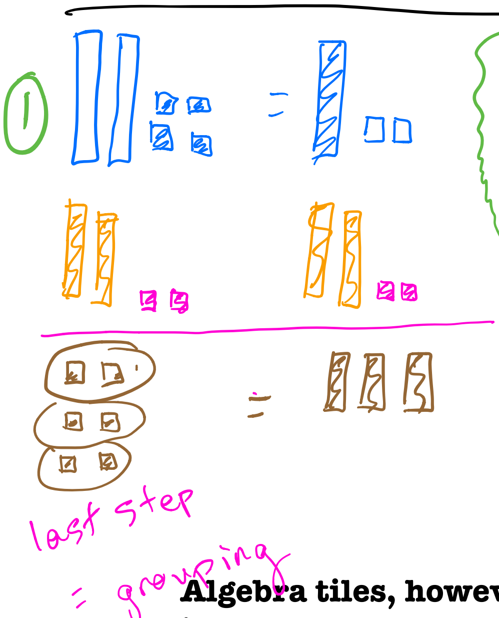
Solving Equations Using Algebra Tiles

To solve an equation using algebra tiles we must isolate the variable tiles and then identify zero pairs.

For example, let's look at the following equation: $-2x + 4 = x - 2$

LS RS

Algebra Tiles	Algebraically
LS = RS	$-2x + 4 = x - 2$
	$4 + 2 = 1x + 2x$
	$\frac{6}{3} = \frac{3x}{3}$
	$2 = x$



LS RS

$-2x + 4 = x - 2$

$+2x \qquad +2x$

$4 = 3x - 2$

$+2 \qquad +2$

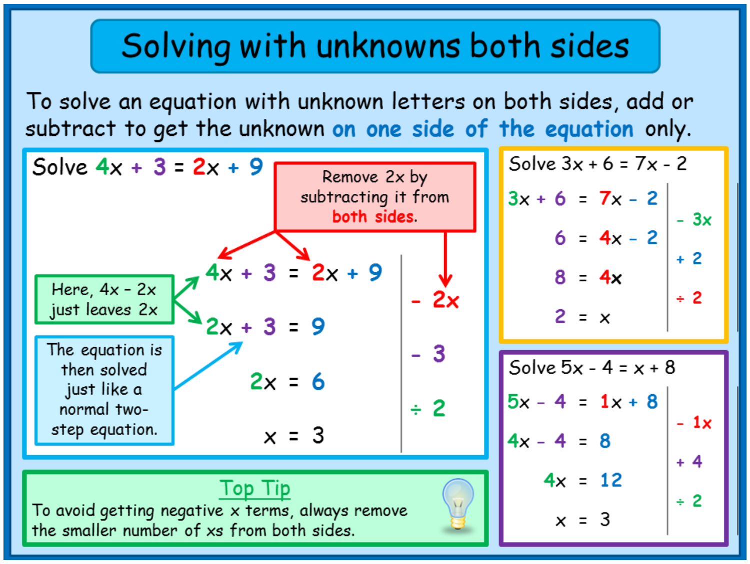
$6 = 3x$

$2 = x$

Algebra tiles, however, is not an efficient method to use when equations involve large number or fractions and/or decimals.

We need to think algebraically!

Solving Equations with Variables on Both Sides



Example:

Solve each of the following algebraically.

***Remember that the goal is to get the variable on one side of the equation and the numbers on the other side of the equation!**

a) $7.4 + 4x = 2x + 13.2$

$4x - 2x = 13.2 - 7.4$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{5.8}{2}$

$\boxed{x = 2.9}$

b) $4x + 7 = 21 - 3x$

$4x + 3 = 21 - 7$

$\frac{7x}{7} = \frac{14}{7}$

$\boxed{x = 2}$

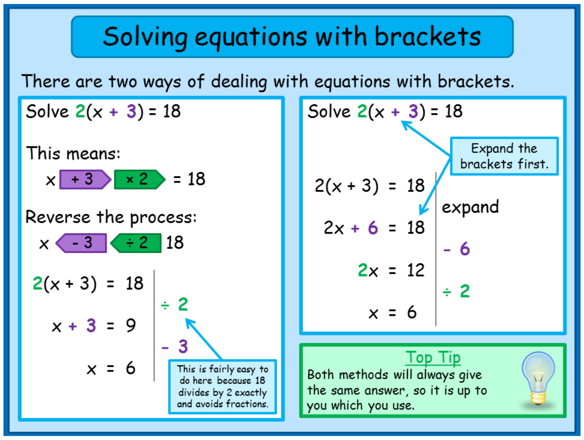
c) $3x + 3 = 5x - 5$

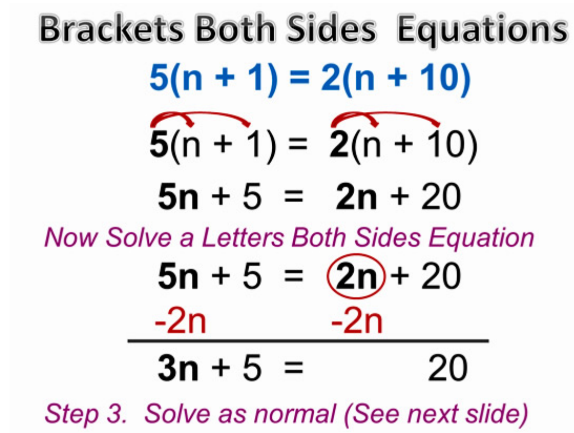
$3 + 5 = 5x - 3x$

$\frac{8}{2} = \frac{2x}{2}$

$\boxed{4 = x}$

Solving Equations with Brackets





Example:

Solve each of the following algebraically.

a) $1.5(x-6)=6$

$\frac{1.5(x-6)}{1.5} = \frac{6}{1.5}$

$x-6=4$

$x=6+4$

$\boxed{x=10}$

b) $3(x-5)=2$

$\frac{3(x-5)}{3} = \frac{2}{3}$

$x-5=\frac{2}{3}$

$x = \frac{2}{3}+5$

$\boxed{x=5\frac{2}{3}}$

c) $4(x-5)=-2(x-2)$

$4x-20=-2x+4$

$\frac{6x}{6}=\frac{24}{6}$

$\boxed{x=4}$ ✓

d) $3(x+1)=5(x-1)$

$3x+3=5x-5$

$\frac{8}{2}=\frac{2x}{2}$

$\boxed{x=4}$

Example:

Two different taxi companies charge the following:

Company A: \$3.00 plus \$0.20 per km ~ $3 + 0.2k$
Company B: \$2.50 plus \$0.25 per km ~ $2.5 + 0.25k$

At what distance will the cost be the same?

a) Model the problem with an equation.

$0.2k + 3 = 0.25k + 2.5$

b) Solve the problem.

$0.2k + 3 = 0.25k + 2.5$
$3 - 2.5 = 0.25k - 0.2k$
$\frac{0.5}{0.05} = \frac{0.05k}{0.05}$
$k = 10$

c) Verify the solution. – you do

$0.2(10) + 3 \stackrel{?}{=} 0.25(10) + 2.5$
$2 + 3 \stackrel{?}{=} 2.5 + 2.5$
$5 = 5$ ✓

Solving Equations with Fractions

The easiest way to solve equations which contain fractions is to **eliminate the denominators**. If we can get rid of all the fractions, the equation will be easier to solve.

We do this by multiplying each term by the whole number you choose. This whole number **must be a common denominator** for all the fractions in the equation.

Solving a Linear Equation Involving Fractions

- Solve the equation:

$$\frac{x}{5} - \frac{1}{2} = \frac{x}{6}$$
$$30\left(\frac{x}{5} - \frac{1}{2}\right) = 30 \cdot \frac{x}{6}$$
$$30 \cdot \frac{x}{5} - 30 \cdot \frac{1}{2} = 30 \cdot \frac{x}{6}$$
$$6x - 15 = 5x$$
$$6x - 5x - 15 = 5x - 5x$$
$$x - 15 = 0$$
$$x - 15 + 15 = 0 + 15$$
$$x = 15$$

- Multiply both sides by the least common denominator 30.
- Be sure to multiply all terms by 30.
- Divide out common factors.
- Subtract 5x to get the x-terms on the left.
- Simplify.

Example:

cover tomorrow

Solve each of the following using algebra.

a) $\frac{x}{9} = \frac{3}{1}$

$x \cdot 1 = 3 \cdot 9$

$x = 27$

b) $\frac{2x}{3} = \frac{4x}{5} + 7$

LCD = 15

$15\left(\frac{2x}{3}\right) = 15\left(\frac{4x}{5} + 7\right)$

$\frac{30x}{3} = \frac{60x}{5} + 105$

$10x = 12x + 105$

$-105 = 2x$

$\frac{2x}{2} = \frac{-105}{2}$

$x = -52.5$

c) $\frac{x}{4} + \frac{1}{5} = \frac{1}{2}$

LCD = 20

$20\left(\frac{x}{4} + \frac{1}{5}\right) = \frac{1}{2}(20)$

$\frac{20x}{4} + \frac{20}{5} = \frac{20}{2}$

$5x + 4 = 10$

$\frac{5x}{5} = \frac{6}{5}$

$x = \frac{6}{5}$

$x = 1\frac{1}{5}$

$x = 1.2$

d) $\frac{1}{2}(x-1) = \frac{2}{3}(1-x)$

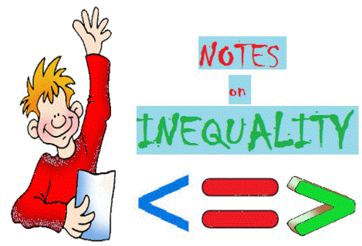
LCD = 6

$\frac{x-1}{2} = \frac{2(1-x)}{3}$

e) $\frac{(2x-3)}{2} = \frac{(-x-1)}{4}$

cross multiply

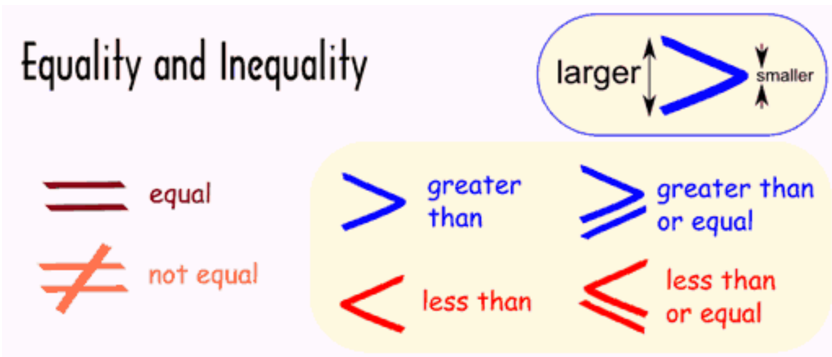
Section 6.3 – Introduction to Linear Inequalities



What are inequalities?

We use inequalities to model a situation that can be described by a range of numbers instead of a single number.

We use specific symbols:



We can use inequalities to represent many real world situations.

For example, which inequality describes the time, *t*, for which a car could be legally parked?

1) $t > 30$

2) $t \geq 30$

3) $t < 30$

4) $t \leq 30$

you can park for 30 mins
∴ 30 is included

Example:

Define a variable and write an inequality for each situation.

a)

$s \leq 55$

b)

$h \geq 102$

c)

$t < 4$

d)

$a \geq 18$

Writing an Inequality to Describe a Situation

Inequalities –

What do they mean in words?

① $x <$
- Less than or smaller than
- Fewer than

② $x \leq$
- Less than or equal to
- No more than
- At most
- A maximum of

③ $x >$
- Greater than or bigger than
- More than

④ $x \geq$
- Greater than or equal to
- No less than
- At least
- A minimum of

Example:

Define a variable and write an inequality to describe each of the following situations.

a) Contest entrants must be at least 18 years old.

$c \geq 18$

b) The temperature has been below −5°C for the last week.

$T < -5$

c) You must have 7 items or less to use the express checkout line at the grocery store.

$i \leq 7$

d) Scientists have identified over 400 species of dinosaurs.

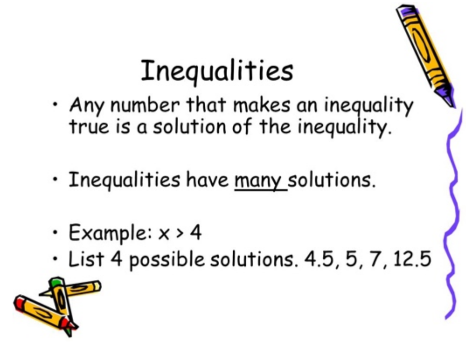
$S > 400$

Determining Whether a Number is a Solution of an Inequality

A **linear equation** is true for only **one** value of the variable.

A **linear inequality** may be true for **many** values of the variable.

The solution of an inequality is any value of the variable that makes the inequality true.



Example:

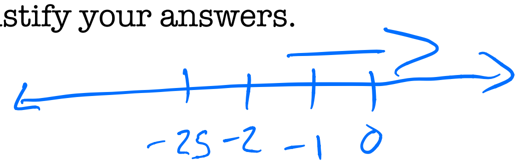
Determine which numbers are a solution of the following inequality.

$$b \geq 3$$

3, 3.01 . . . . . .

Example:

Is each number a solution to the inequality $x > -2$? Justify your answers.



a) ~~-8~~

b) ~~-2~~

c) 0 ✓ 0 is greater than -2

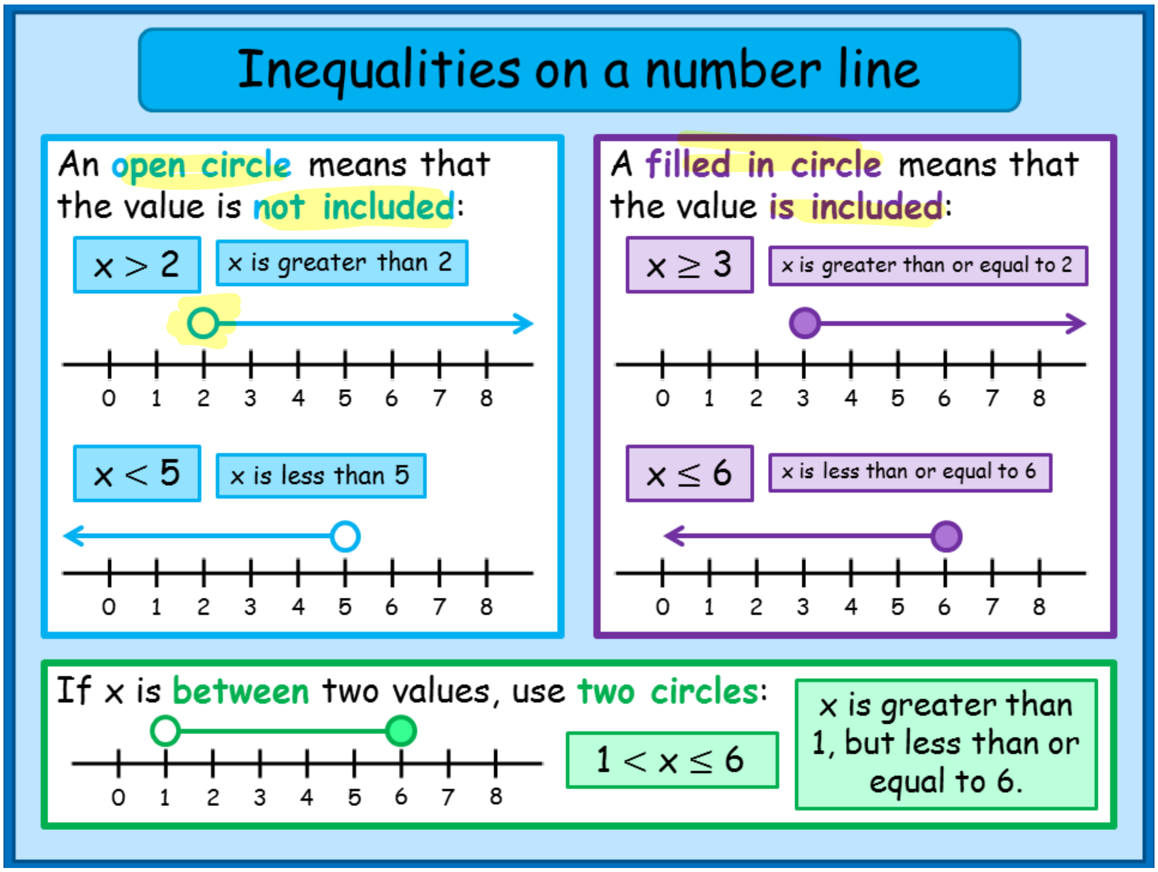
d) 2 ✓ 2 is greater than -2

e) ~~-2.5~~

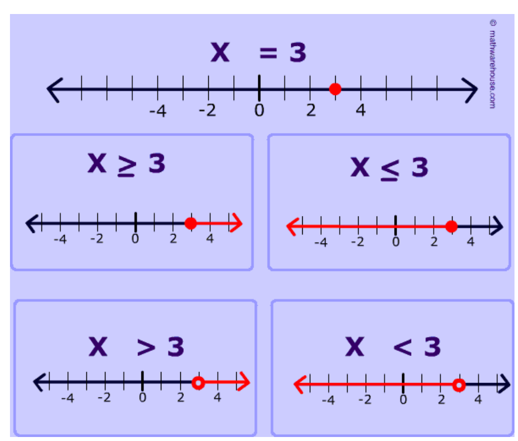
f) ~~-3.5~~

Graphing Inequalities on a Number Line

Since the solution of an inequality is **any value** of the variable that makes the inequality true, there are usually too many numbers to list. So we can show them on a number line.



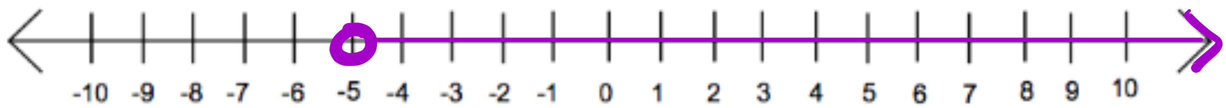
Symbol	Meaning	Closed or Open Circle
<	Less Than	Open ○
>	Greater Than	Open ○
≤	Less Than or Equal to	Closed ●
≥	Greater Than or Equal to	Closed ●



Example:

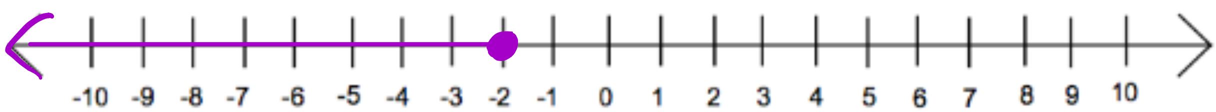
Graph each inequality on a number line and list 4 numbers that are solutions on the inequality.

a) $t > -5$

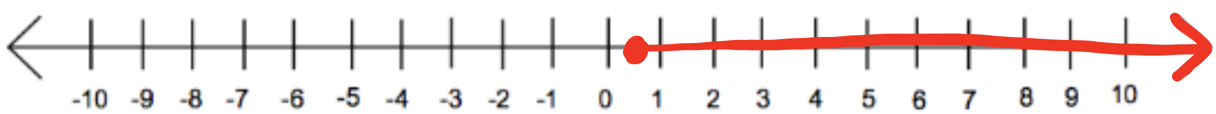


-4.99 $3, 10, 45$

b) $-2 \geq x$



c) $0.5 \leq a$



d) $p < \frac{-24}{3}$

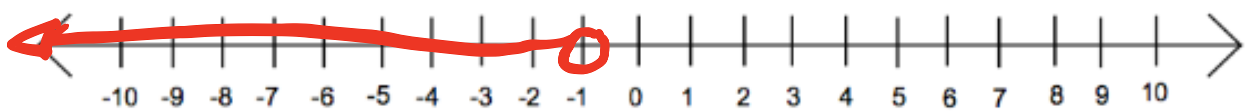
Example:

Write an inequality to describe each situation, then graph the solution on a number line.

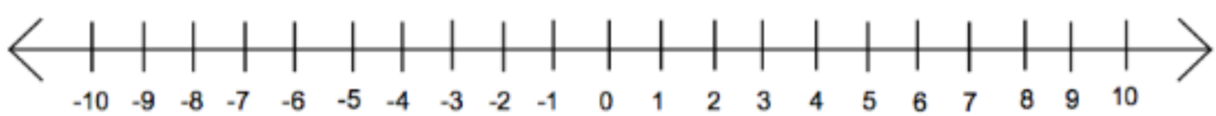
a) A number is greater than 5. $x > 5$

b) A number is less than or equal to 4.

c) The temperature is below -1°C today. $T < -1$

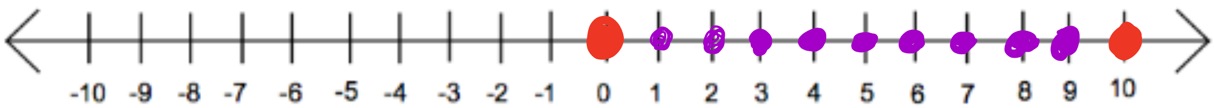


d) In most provinces, you have to be at least 16 years old to drive.



$A \geq 16$

e) You must have 10 items or less to go to the express lane at the grocery store.



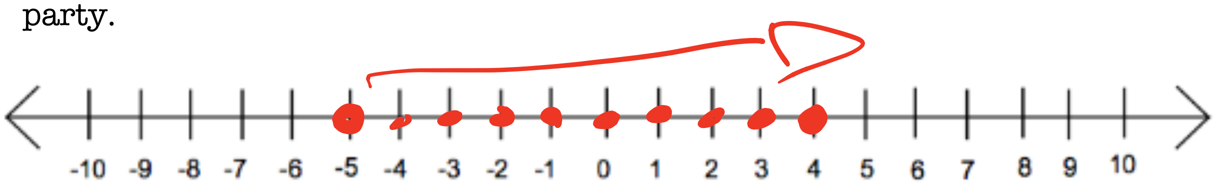
Note: Any **whole number** greater than or equal to 10, but not greater than 10 has to be included in the answer.

This problem involves **discrete data**.

This time decimals or fractions are NOT included, due to the situation. (You can't buy half an item!)

We represent it on a number line by putting **solid dots** on each included possible number.

f) Sarah's mom said she should invite at least 10 people to her birthday party.



Summary

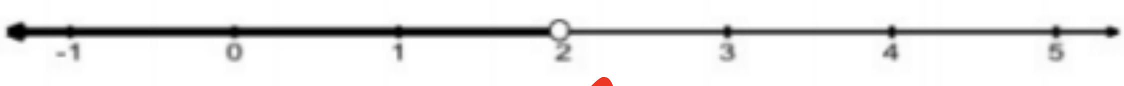
When graphing inequalities:

- $>$ or $<$ use hollow dots on the number line
- $\geq$ or $\leq$ use solid dots on the number line

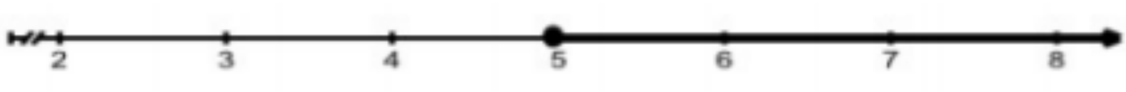
- if continuous data, shade the line
- if discrete data, use dots on whole numbers

Example:

Write the inequality for each of the following number lines.

a) 

$x < 2$

b) 

$x \geq 5$

c) 

$x \leq -1$

d) 

$-6 < x$

Section 6.4 – Solving Linear Inequalities Using Addition & Subtraction

Let's Investigate!!

Part A: Add a Positive and Negative Number

Operation	$-4 < -2$	$6 > 2$
ADD a **positive** number to each side of the inequality	$-3 < -1$	$7 > 3$
ADD a **negative** number to each side of the inequality	$-5 < -3$	$5 > 1$

Do the inequalities hold true? yes!

Part B: Subtract a Positive and Negative Number

Operation	$-4 < -2$	$6 > 2$
SUBTRACT a **positive** number to each side of the inequality	$-5 < -3$	$5 > 1$
SUBSTARCT a **negative** number to each side of the inequality	$-3 < -1$	$7 > 3$

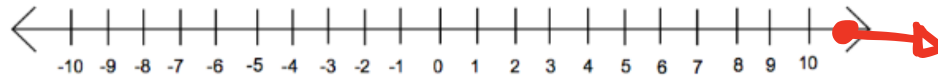
Do the inequalities hold true? yes!

When adding or subtracting to solve an inequality, it is the exact same as solving an equation.

Example:

Solve each of the following inequalities, verify the solution and graph on a number line.

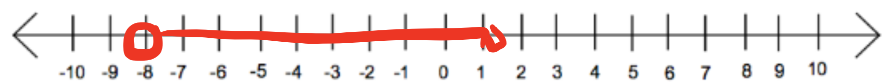
a) $6.2 \leq x - 4.5$



$6.2 + 4.5 \leq x$ → $6.2 = 10.7 - 4.5$

$10.7 \leq x$ → $6.2 = 6.2$ ✓

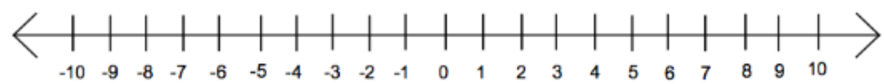
b) $4.8 + x > -3.2$



$x > -3.2 - 4.8$ → $4.8 + -8 = -3.2$

$x > -8$ → $-3.2 = -3.2$

c) $x + 4 < 7$



$x < 3$ → $3 + 4 = 7$

$7 = 7$ ✓

Example:

Jack plans to board his dog while he is away on vacation.

$d =$ days

- Boarding House A charges \$90 plus \$5 per day
- Boarding House B charges \$100 plus \$4 per day

$C_A = 90 + 5d$

$C_B = 100 + 4d$

For how many days must Jack board his dog in House A to be less expensive than House B?

a) Choose a variable and write an inequality.

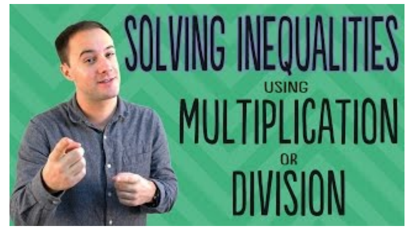
$90 + 5d < 100 + 4d$

b) Solve the problem.

$5d < 10 + 4d$ → $d < 10$

c) Graph the solution.

Section 6.5 – Solving Linear Inequalities Using Multiplication & Division



Let's Continue to Investigate!!

Part C: Multiply by a Positive and Negative Number

Operation	$-4 < -2$	$6 > 2$
MULTIPLY each side of the inequality by a **positive** number	$2 \cdot -4 < -2 \cdot 2$ $-8 < -4$ true	$2 \cdot 6 > 2 \cdot 2$ $12 > 4$ true
MULTIPLY each side of the inequality by a **negative** number	$-2 \cdot -4 < -2 \cdot -2$ $8 < 4$ false	$-2 \cdot 6 > 2 \cdot -2$ $-12 > -4$ false

Do the inequalities hold true? Depends

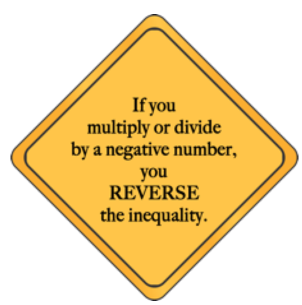
Part D: Divide by a Positive and Negative Number

Operation	$-4 < -2$	$6 > 2$
DIVIDE each side of the inequality by a **positive** number	$\frac{-4}{2} < \frac{-2}{2}$ $= -2 < -1$ true	$\frac{6}{2} > \frac{2}{2}$ $3 > 1$ true
DIVIDE each side of the inequality by a **negative** number	$\frac{-4}{-2} < \frac{-2}{-2}$ $2 < 1$ false	$\frac{6}{-2} > \frac{2}{-2}$ $-3 > -1$ false

Do the inequalities hold true? Depends

Solving inequalities is the exact same as solving an equation with **one exception**:

When MULTIPLYING OR DIVIDING an inequality by a NEGATIVE number ⟶ REVERSE the inequality symbol



Example:

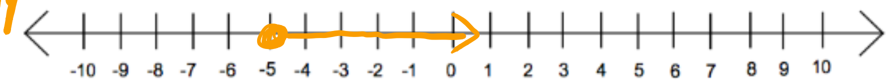
Solve each of the following inequalities, verify the solution and graph on a number line.

flip inequality

a) $-5x \leq 25$

$\frac{-5x}{-5} \quad \frac{25}{-5}$

$x \geq -5$



b) $7a < -21$

$\frac{7a}{7} \quad \frac{-21}{7}$

$a < -3$

c) $\frac{x}{-4} > -3 \quad \cdot -4$

$x < 12$

Example:

Solve and verify: $-2.6x + 14.6 > -5.2 + 1.8x$

$-2.6x - 1.8x > -5.2 - 14.6$

$\frac{-4.4x}{-4.4} > \frac{-19.8}{-4.4}$

$x < 4.5$

Example:

A super slide charges $1.25 to rent a mat and $0.75 per ride. James has $10.25. How many rides can James go on?

r = ride

a) Choose a variable and write an inequality.

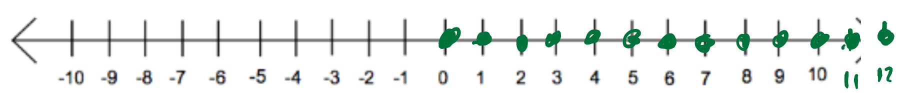
$1.25 + 0.75r \leq 10.25$

b) Solve the problem.

$0.75r \leq 10.25 - 1.25$

$0.75r \leq 9$

c) Graph the solution.



$\frac{0.75r}{0.75} \leq \frac{9}{0.75}$

$r \leq 12$ James can go on 12 or less rides

Summary for Solving Inequalities

Rules for Solving Inequalities:

1. Make the same changes to both sides of the inequality
2. Isolate the variable
3. Combine Like Terms
4. Use the Inverse Operation to remove clutter away from variable
5. BUT, if your Inverse Operation is multiplication or division by a *negative number*, the inequality sign reverses

$<$ becomes $>$
$>$ becomes $<$
$\le$ becomes $\ge$
$\ge$ becomes $\le$

Solving Inequalities

Solving Two-Step Inequalities

1. Add or subtract to isolate the variable term.
2. Multiply or divide to solve for the variable. If multiply or divide by a negative number then reverse the inequality symbol.

Example:

$$-3x + 5 \le -16$$

$$-5 \qquad -5 \quad \text{Subtract}$$

$$-3x \le -21$$

$$\frac{-3x}{-3} \ge \frac{-21}{-3} \quad \text{Divide by -3, reverse inequality}$$

$$x \ge 7$$

An equation has only one solution, while an inequality has a range of solutions.

Solve an Equation	Solve an Inequality
$7x = 2x + 15$ $7x - 2x = 2x - 2x + 15$ $5x = 15$ $\frac{5x}{5} = \frac{15}{5}$ $x = 3$ only one answer for x	$7x < 2x + 15$ $7x - 2x < 2x - 2x + 15$ $5x < 15$ $\frac{5x}{5} < \frac{15}{5}$ $x < 3$ more than one answer for x (a range of answers)

Verify Equation	Verify Inequality
$7x = 2x + 15$ $7(3) = 2(3) + 15$ $21 = 6 + 15$ $21 = 21$ ✓	$7x < 2x + 15$ Since the solution says x < 3, choose any value and substitute in for x. Try $x = 2$ $7(2) < 2(2) + 15$ $14 < 4 + 15$ $14 < 19$ ✓

Extra Examples:

Solve each inequality and graph the solution on a number line.

a) $3x + 1 > 10$

b) $-3(x - 2) \leq 9$

$-3x + 6 \leq 9$

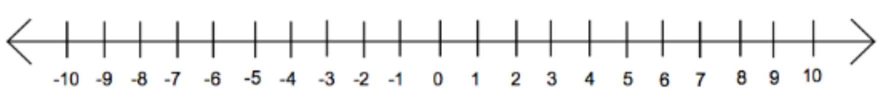
$\frac{-3x}{-3} \leq \frac{3}{-3}$

$x \geq -1$

c) $3x + 1 > 4x - 2$

$3 > x$

d) $3x \geq \frac{3}{5}(x - 2)$



$5 \cdot 3x \geq \frac{3(x-2)}{5} \cdot 5$

$15x \geq 3(x - 2)$

$x \geq -\frac{1}{2}$

$15x \geq 3x - 6$

$\frac{12x}{12} \geq \frac{-6}{12}$

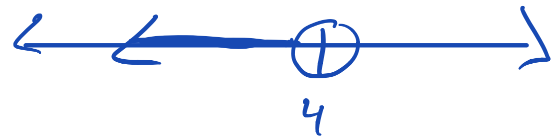
Word Problems: Linear Equations & Inequalities

1. Write an equation or inequality for each statement below and solve. Show the solutions for all inequalities on a number line.

a) Triple a number decreased by one is less than 11.

$3x - 1 < 11$

$x < 4$



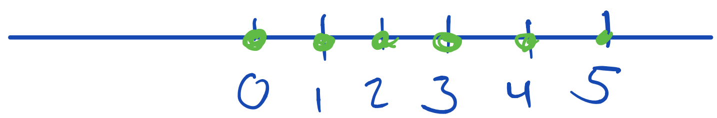
b) A number multiplied by 4, increased by 5 is 1.

$4x + 5 = 1$

$x = -1$

c) You can invite at most 5 friends over to your house Saturday evening.

$x \leq 5$



d) Five subtract 3 times a number is equal to 3.5 times the same number subtract eight.

$5 - 3x = 3.5x - 8$

$5 + 8 = 3.5x + 3x$

$\frac{13}{6.5} = \frac{6.5x}{6.5}$

$\rightarrow x = 2$

e) Sam has a choice of two companies to rent a car.
Company A charges \$199 per week plus \$0.20 per km driven.
Company B charges \$149 per week plus \$0.25 per km driven.
At what distance will both companies cost the same?

$\underbrace{199 + 0.20k}_{A} = \underbrace{149 + 0.25k}_{B}$

$k = ?$

$199 + 0.20k = 149 + 0.25k$

$0.20k - 0.25k = 149 - 199$

$\frac{-0.05k}{-0.05} = \frac{-50}{-0.05}$

K = 1000 kilometers

2. For the inequality $2(5 - 3x) \geq -7x + 2$, Kate says the solution is $x \geq -8$. Choose values to verify whether or not this is correct.

3. Michael is 7 years younger than his sister, Sarah. How old must each be if the sum of their ages is greater than 25?

$M = S - 7$

$S + M > 25$

$S + (S - 7) > 25$

$2s - 7 > 25$

$2s > 32$

$S = 16$

4. Jenna rents a car for \$350 plus \$12.50 per day on her vacation. If she budgeted \$900 for her car rental, for how many days can she rent the car? Graph the solution on a number line.

$350 + 12.50d \leq 9000$

$12.50d \leq 900 - 350$

$\frac{12.50d}{12.50} \leq \frac{550}{12.50}$

$d \leq 44$