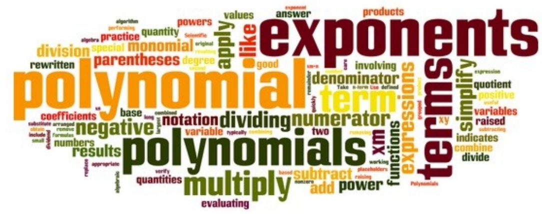
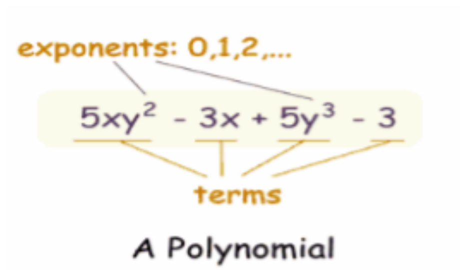
## Section 5.1 – Modelling Polynomials



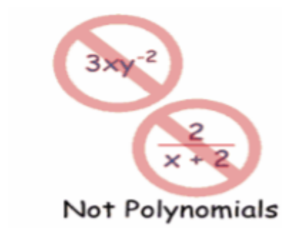

A **polynomial** is an algebraic expression that contains one term or the sum of terms.

The term(s) may contain variables (which will have whole number (positive) exponents).

A term may be a number.




**Note:** An algebraic expression that contains a term with a **variable in the denominator**, a **negative exponent** or the **square root of a variable** is **NOT** a polynomial.




For example, $3x + 1$ is a polynomial.

It contains a variable (whose exponent is 1) and numbers.
Since it is an **expression**, there is no equal sign.

This polynomial has two **terms**. A term is a number, or a variable, or the product of numbers and variables. Terms are separated by + or − . Therefore, $3x$, is one term and 1, is another term.

In the term, $3x$, the 3 is called the **numerical coefficient**. This is the number in front of the variable, it's the numerical factor of a term. The $x$ is called the **variable**.

1 is called the **constant term**. There is no variable attached to this number. It is the number in the expression that does not change.

## Types of Polynomials

Different kinds of polynomials

- What does "poly" mean?
  - So a polynomial has MANY terms
- What number does "mono" represent?
  - So a monomial has ONE term
- What number does "bi" represent?
  - So a binomial has TWO terms
- What number does "tri" represent?
  - So a trinomial has THREE terms

We can classify a polynomial by the numbers of terms it has. Polynomials with 1, 2, or 3 terms have special names.

A **monomial** has 1 term; for example: $5x$ , $9$ , $-2p^2$
A **binomial** has 2 terms; for example: $2c - 5$ , $2m^2 + 3m$ , $x + y$
A **trinomial** has 3 terms; for example: $2h^2 - 6h + 4$ , $x + y + z$

**Example:**

For each of the following polynomials identify:

(i) the variable
(ii) the number of terms
(iii) the numerical coefficient(s)
(iv) the constant term
(v) the type of polynomial

a). $3x^2 + 2x - 1$ b). $6xy - x^3$ c). $xy + 6 - z + 2x^2$

## Equivalent Polynomials

Polynomials that have exactly the same terms, but the terms could be in a different order.

$3x^2 + 2x - 1$ is equivalent to $2x + 3x^2 - 1$ but not equivalent to $1 + 2x - 3x^2$

**Why?**

Both these polynomials have
+3 with $x^2$
+2 with $x$ and
a constant term of – 1

This polynomial is different because
it has – 3 with $x^2$ and
a constant term of +1

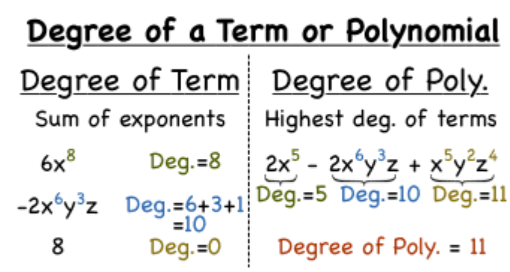
**Example:**

Are the following polynomials equivalent? Explain why or why not.

a) $3x^2 + 2x - 1$ and $2x + 3x^2 - 1$

b) $3x^2 + 2x - 1$ and $1 + 2x - 3x^2$

<u>**Degree of a Polynomial**</u>




**Degree:** The term with the greatest exponent.

***Rules for determining the degree:***

- The **degree of a monomial** is the sum of the exponents of its variables.

| Monomial | Degree |
|---|---|
| $4x^2$ | 2 |
| $9a^2b^3c$ | 6 |

- The **degree of a polynomial with one variable** is the highest power of the variable in any one term.

| Polynomials | Degree |
|---|---|
| $6x^2 + 3x$ | 2 |
| $x^9 + 7x^2 - 3$ | 9 |

- The degree **of a polynomial in two variables or more** is the largest sum of the variables in any one term.

| Polynomials | Degree |
|---|---|
| $x^2y^3 + xy^4 + xy^5$ | 6 |
| $3x^3y^4 + 7xy^3 - 2xy$ | 7 |

**Example:**

Name the coefficients, degree and the constant term of each polynomial.

a) $-3x^2 + 4x - 5$ b) $8 + 7ab + a^2b^2$ c) $-6 - 5x$

**Example:**

State the degree of each polynomial.

a) $9x^2y^2 + 3xy^3$ b) $3x + 2y - 3z$ c) $29n^3x + 36n^3x^3$

A polynomial should be written in **descending order**. This means the exponent of the variable should decrease from left to right.

For example, the polynomial $2k - 4k^2 + 7$ is properly written as $-4k^2 + 2k + 7$ in descending order.

**Example:**

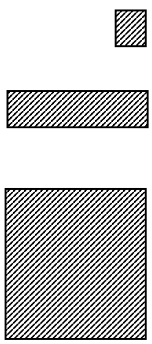
Rearrange the following polynomials in descending order.

a) $-2p + 4p^2 - 9$ b) $5x^2 + 7 - 8x$ c) $33 + 90c + 100c^2$

## Modeling Polynomials

In algebra we use algebra tiles to model integers and variables.

Shaded tiles represent **negative** tiles

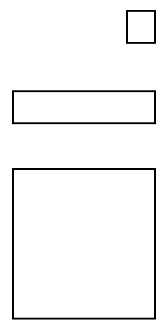


$= -1$

$= -x$

$= -x^2$

Non-shaded tiles represent **positive** tiles



$= +1$

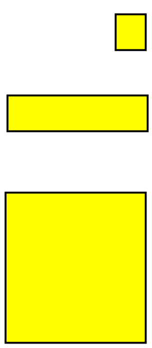
$= +x$

$= +x^2$

Colors can also be used to represent a tile.

**IN YOUR TEXTBOOK**:

Yellow is Positive

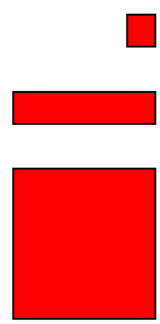


$= +1$

$= +x$

$= +x^2$

Red is Negative



$= -1$

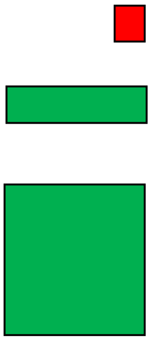
$= -x$

$= -x^2$

The variable most commonly used is $x$, however, any variable can be used.

**REAL ALGEBRA TILES**:

Green and Red is Positive

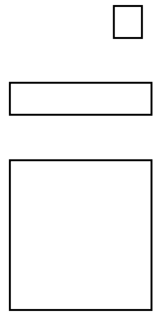


= $+1$

= $+x$

= $+x^2$

White is Negative

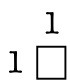


= $-1$

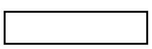
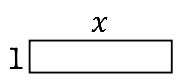
= $-x$

= $-x^2$

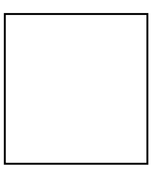
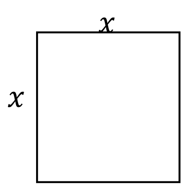
Algebra tiles get their name from the **area** of their tiles.
Remember length × width = area

 = $+1$ because 

the length and width of the tile is 1 and $1 \times 1 = 1$

 = $+x$ because 

the length is $x$ and the width is 1 and $1 \times x = x$

 = $+x^2$ because 

the length and width of the tile is $x$ and $x \times x = x^2$

**Example**:

Use algebra tiles to model each expression.

a) $3x^2 - 2x + 5$

b) $x^2 + 3x - 6$

c) $2b^2 - b + 4$

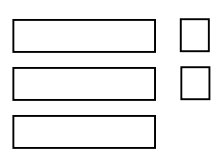
d) $5a - 3$

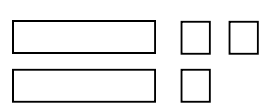
**Example:**

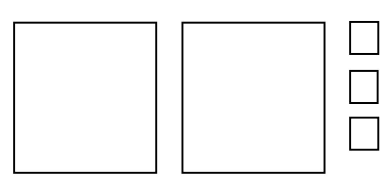
Match each of the following polynomials to the appropriate diagram.

A) $2x^2 + 3$

B) $3x + 2$

i) 

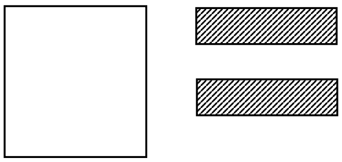
ii) 

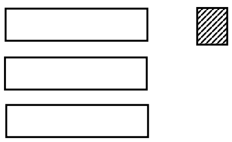


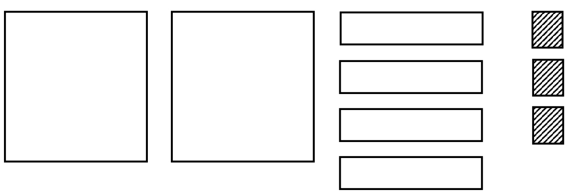
C) $2x + 3$ iii)

**Example:**

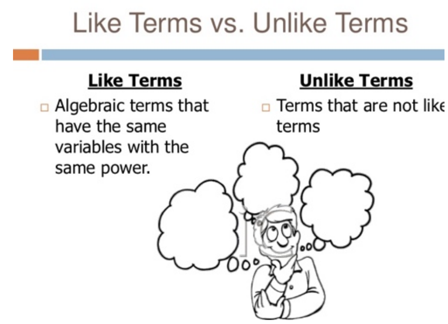
Write a polynomial expression for each diagram.

a) 

b) 

c) 

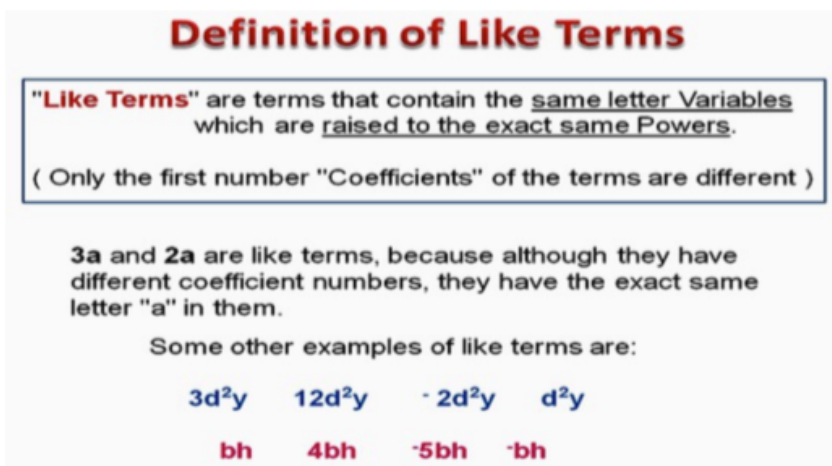
## Section 5.2 – Like & Unlike Terms

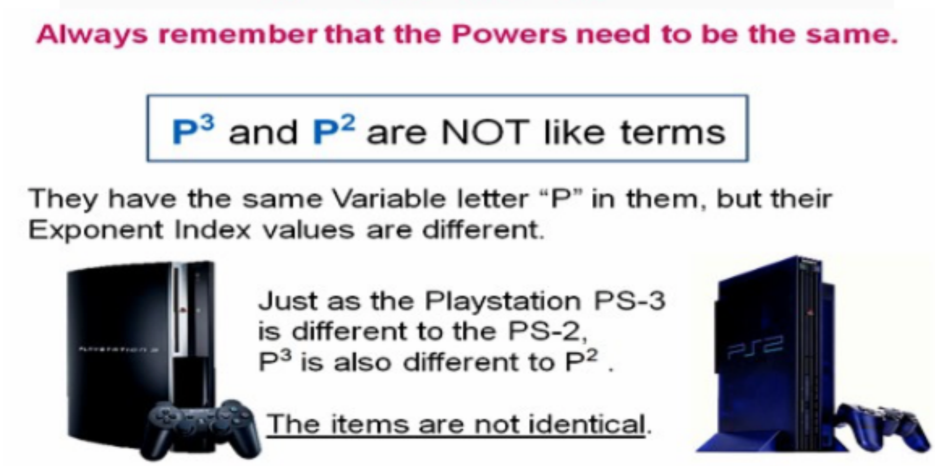



A polynomial is in **simplified form** when:

- its algebra tile model uses the fewest tiles possible
- its symbolic form contains only one term of each degree and no terms with a zero coefficient

Terms that can be represented by algebra tiles with the same size and shape are called **like terms**.



## Algebra – Like n Unlike terms

Observe 2x, 4x, 23x, 51x ..
These *algebraic terms* are having similar *literal coefficient* i.e. x
We call such similar looking *algebraic terms* as

**Like terms**

Example:
1) 5y, 9y, 13y ➡ *It is having same coefficient **y***
2) 4m, m, 2m, 18m ➡ *Here coefficient is **m** for all*

**Like terms looks alike and similar**

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## Algebra - Like n Unlike terms

3x, 8y, 34c, 423z .. Are *algebraic terms*, having *different coefficients* x, y, c, z
So we call such *algebraic terms* as

**Unlike terms**

Example:
1) 2y, 19z, 23a ➡ *These are having different coefficients*
2) ma, 3a, 22c, 18x ➡ *Here coefficients are **not same** for all*

**Unlike terms looks differently**

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| Like Terms | Unlike Terms | Why are they Unlike Terms? |
|---|---|---|
| $2x + 19x$ | $2x + 19a$ | The variables are different. |
| $4w - 10w$ | $4w - 10w^2$ | The exponents are different. |
| $14.2r - 12r$ | $12r - 12s$ | The variables are different. |
| $32a^2 + 9a^2$ | $32a^2 + 9a^3$ | The exponents are different. |
| $8y + 5y$ | $8y + 5$ | One term is a constant and the other has a variable. |

## Like Terms - EXAMPLES

Decide if the terms in each pair of items are "Like Terms".

1) $4g$ and $4h$ ______
2) $3h$ and $^-h$ ______
3) $5x$ and $4xy$ ______
4) $2x^2y^3$ and $2x^2y^5$ ______
5) $5p^2q^3$ and $^-4p^2q^3$ ______

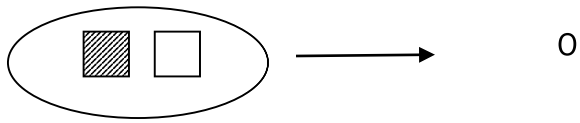
## Like Terms - ANSWERS

Decide if the terms in each pair of items are "Like Terms".

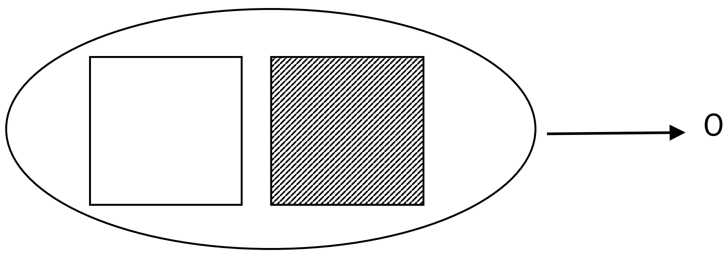
1) $4g$ and $4h$ NO – letter variables are different.
2) $3h$ and $^-h$ YES – letters the same ( $^-h = {}^-1h$)
3) $5x$ and $4xy$ NO – letter variables are different.
4) $2x^2y^3$ and $2x^2y^5$ NO – y powers are different.
5) $5p^2q^3$ and $-4p^2q^3$ YES – letters & powers same

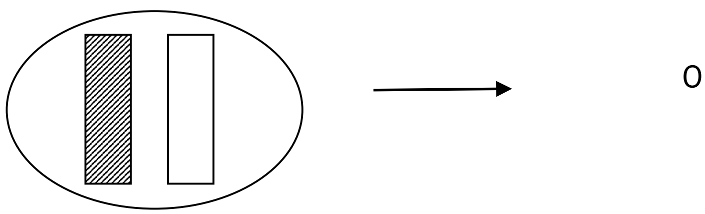
## Simplifying Polynomials Using Algebra Tiles

When we worked with integers, a +1 tile and a -1 tile formed a **zero pair**.




The same applies for the $x$ and $x^2$ tiles.




Any two **opposite colored tiles** of the **same size** have a sum of **zero**. We can combine these tiles because they are **like terms**.

To simplify a polynomial using algebra tiles, like terms can be combined and zero pairs removed.

**Example:**

Sketch the tiles and cancel the zero pairs where possible, to simplify the polynomials listed below.

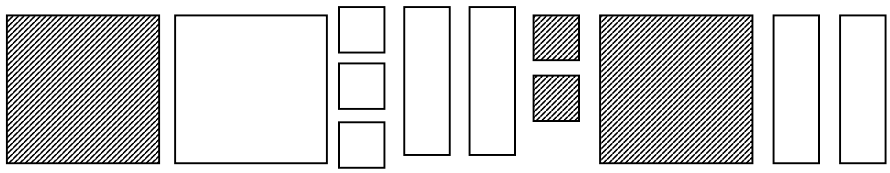
a) $4x$ and $-2x$

b) $+1$ and $+8$

c) $x^2$ and $-3x^2$

**Example:**

Write a simplified expression for the algebra tiles below.



**Example:**

Sketch the simplified expression using algebra tiles for:

$$4n^2 - 1 - 3n - 3 + 5n - 2n^2$$

Can you see how to simplify like terms without using tiles?

$4n^2 - 1 - 3n - 3 + 5n - 2n^2$ means $4n^2$ and $-2n^2 = -2n^2$

$-1 - 3n - 3 + 5n$ means $-3n$ and $5n = 2n$

$-1 - 3$ means $-1$ and $-3 = -4$

*It's just like adding integers. Be careful of the signs!!!!

Answer: $2n^2 + 2n - 4$

## Simplifying Polynomials By Combining Like Terms

We can also simplify a polynomial by adding the coefficients of like terms. This is called **combining like terms**.

### Steps for Combining Like Terms

**To Combine Like Terms, follow these steps:**

- Identify the items which are "Like Terms"
- Rewrite the expression so that the like terms are all next to each other
- Combine the groups of like terms together to make a simplified shorter final answer

This last step involves adding or subtracting the like terms

**Like Terms – Example One**

Simplify: $7mn - 2mn + 3mn$

$7mn - 2mn + 3mn$ (three like terms)

$= 5mn + 3mn$

$= 8mn$

$= \mathbf{8mn}$ ✓

**Like Terms – Example Two**

Simplify : $4g + 3h + 2g + 3gh + 6hg$

$4g + 3h + 2g + 3gh + 6gh$ ( $6hg = 6gh$ )

$= 4g + 2g + 3h + 3gh + 6gh$

$= 6g + 3h + 9gh$

$= 6g + 3h + 9gh$ ✓

**Like Terms – Example Three**

Simplify the expression: $4w + 3 + 2w - 1$

$4w + 3 + 2w - 1$ (Now Group Like Terms)

$= 4w + 2w + 3 - 1$ (Combine Like Terms)

$= 6w + 2$

$= \mathbf{6w + 2}$ ✓

**Like Terms – Example Four**

Simplify: $2a^3 - 10ab^2 + 3a^3 - ab^2 - 7$

$2a^3 - 10ab^2 + 3a^3 - ab^2 - 7$

$= 2a^3 + 3a^3 - 10ab^2 - 1ab^2 - 7$

$= 5a^3 - 11ab^2 - 7$

$= \mathbf{5a^3 - 11ab^2 - 7}$ ✓

**Like Terms – Example Five**

Simplify the expression: $4a^2 + 3a + 5a^3 - 1$

The expression contains terms that are all different from each other.

The expression cannot be simplified any further.

$\mathbf{4a^2 + 3a + 5a^3 - 1}$ ✓

**Example:**

Simplify each polynomial **without using algebra tiles**.

a) $3x + 5x$

b) $-13a - 10a$

c) $16n + n - 17n$

d) $-j + 7k - 3j$

e) $8a - 2b - 6a - 3b$

f) $-q + 7q + 11n + 11p - 8q$

**Example:**

Mike was asked to write an expression equivalent to $2x - 7 - 4x + 8$. Below are his workings.

His solution was:
$$2x - 7 - 4x + 8$$
$$= 2x - 4x - 7 + 8$$
$$= 2x - 1$$

What errors did he make? Show the correct simplification.

## Section 5.3 – Adding Polynomials

To add polynomials, we just need to combine like terms.

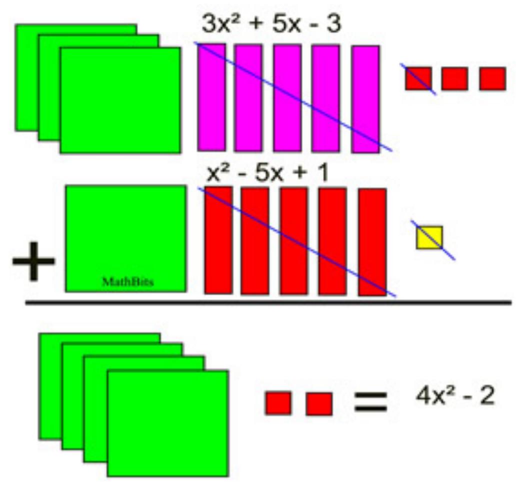

### Adding Polynomials Using Algebra Tiles

To add polynomials using algebra tiles, we combine the algebra tiles that represent each polynomial, group like terms, then remove all zero pairs. The remaining tiles represent the polynomial that is the answer.

For example, to determine the sum of $3x^2 + 5x - 3$ and $x^2 - 5x + 1$, we would write:

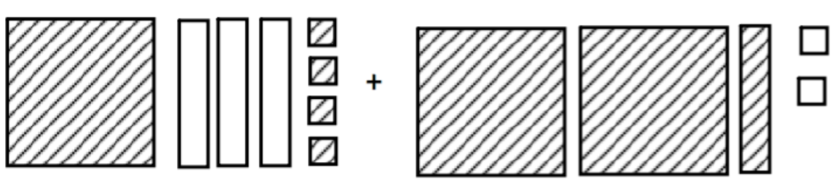
$$(3x^2 + 5x - 3) + (x^2 - 5x + 1)$$

- Using tiles, display both polynomials
- Remove zero pairs
- The remaining tiles represent: $4x^2 - 2$




**Example:**

Add using algebra tiles.

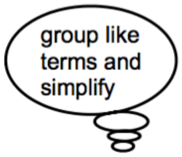


## Adding Polynomials Symbolically (Using Algebra)

To add polynomials symbolically, we combine like terms by adding the coefficients of like terms.

Polynomials can be added horizontally or vertically.

**Horizontally**




**1. Using a horizontal method to add like terms:**

Remove parentheses. Identify like terms. Group the like terms together. Add the like terms.

$(2x^2 - 4) + (x^2 + 3x - 3)$

$= 2x^2 - 4 + x^2 + 3x - 3$ .. identify like terms

$= 2x^2 + x^2 + 3x - 4 - 3$ .. group the like terms together

$= 3x^2 + 3x - 7$ ... add the like terms

**Adding Polynomials**

**Add: $(x^2 + 3x + 1) + (4x^2 + 5)$**

**Step 1: Underline like terms:**

$(\underline{x^2} + 3x + \underline{\underline{1}}) + (\underline{4x^2} + \underline{\underline{5}})$

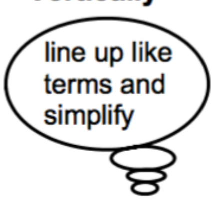
*Notice: '3x' doesn't have a like term.*

**Step 2: Add the coefficients of *like terms*, do not change the powers of the variables:**

$(x^2 + 4x^2) + 3x + (1 + 5)$

$\boxed{5x^2 + 3x + 6}$

**Vertically**




**2. Using a vertical method to add like terms:**

Arrange the like terms so that they are lined up under one another in vertical columns, adding 0 place holders if necessary. Add the like terms in each column following the rules for adding signed numbers.

$$\begin{array}{r} 2x^2 + 0x - 4 \\ + \; x^2 + 3x - 3 \\ \hline 3x^2 + 3x - 7 \end{array}$$

**Adding Polynomials**

Some people prefer to add polynomials by stacking them.
If you choose to do this, be sure to line up the like terms!

$(x^2 + 3x + 1) + (4x^2 + 5) \Rightarrow$

$$\begin{array}{r} (x^2 + 3x + 1) \\ + \; (4x^2 \qquad + 5) \\ \hline \boxed{5x^2 + 3x + 6} \end{array}$$

Stack and add these polynomials: $(2a^2+3ab+4b^2) + (7a2+ab+-2b^2)$

$(2a^2+3ab+4b^2) + (7a^2+ab+-2b^2) \Rightarrow$

$$\begin{array}{r} (2a^2 + 3ab + 4b^2) \\ + \; (7a^2 + ab + -2b^2) \\ \hline \boxed{9a^2 + 4ab + 2b^2} \end{array}$$

**Example:**

Add symbolically (using algebra).

a) $(-2x^2 - 3x) + (2x + x^2)$

b) $(7n + 14) + (-6n^2 + n - 6)$

c) $(3x^2 + 2x + 4) + (-5x^2 + 3x - 5)$

**Example:**

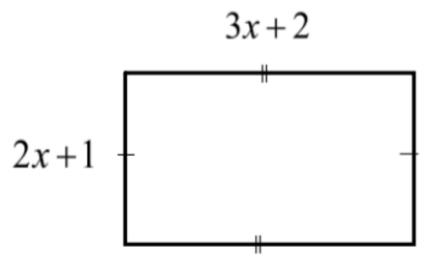
Add the following both horizontally and vertically.

$$(2x^2 + 3x - 2) + (-x^2 + 7x - 3)$$

<u>**Horizontally**</u> <u>**Vertically**</u>

**Example:**

Write a polynomial for the perimeter of this rectangle.




Remember each side of the rectangle is there twice

**Example:**

Add the following polynomials.

$$(2a^2 + a - 3b - 7ab + 3b^2) + (-4b^2 + 3ab + 6b - 5a + 5a^2)$$

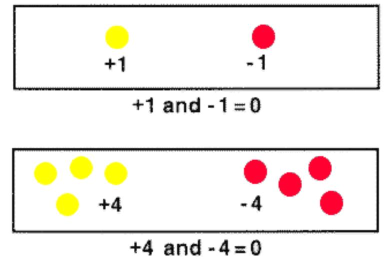
**Example:**

A student added $(4x^2 - 8x + 1) + (2x^2 - 6x - 2)$ as follows:

$$\begin{aligned}
&(4x^2 - 8x + 1) + (2x^2 - 6x - 2) \\
&= 4x^2 - 8x + 1 + 2x^2 - 6x - 2 \\
&= 4x^2 + 2x^2 - 8x - 6x + 1 - 2 \\
&= 6x^2 - 2x - 1
\end{aligned}$$

Is the student's work correct? If not, explain where the student made any errors and write the correct solution.

## Section 5.4 – Subtracting Polynomials




To subtract polynomials, we just need to **add the opposite**.

Recall, opposite numbers have a sum of zero.

**Example:**

What is the opposite of each of the following?

a) $2.4$ b) $-10$

The same idea applies to polynomials. Opposite polynomials will have a sum of zero.

For example, the opposite of $2x$ is __________ and the opposite of $-x^2$ is __________.

**Example:**

What is the opposite of each polynomial below?

a) $11$

b) $-5x$

c) $-24x^4$

Getting the opposite of a monomial is just like getting the opposite of a number.

**Think About This??**

How do we get the opposite of a binomial or trinomial?

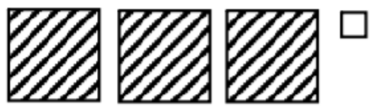
a) $2x + 3$

b) $4x^2 - 7x + 3$

c) $-2xy - 2y^2 + 3x^2$

**Example:**

Sketch the opposite of the polynomial using algebra tiles.

a) 

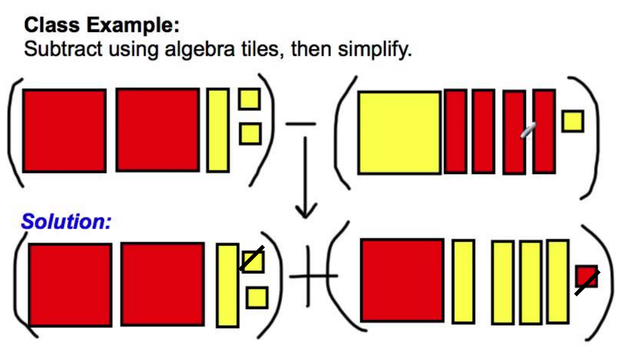
b) 

## Subtracting Polynomials Using Algebra Tiles

To subtract polynomials using algebra tiles, we add the opposite by rewriting the expression. Then we combine the algebra tiles that represent each polynomial, group like terms, then remove all zero pairs. The remaining tiles represent the polynomial that is the answer.

For example, to determine the difference of $-2x^2 + x + 2$ and $x^2 - 4x + 1$, we would write:

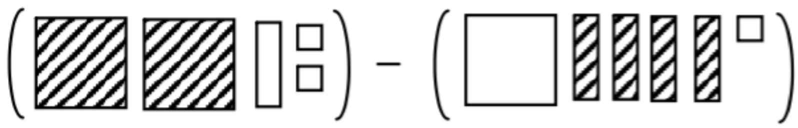
$$(-2x^2 + x + 2) - (x^2 - 4x + 1) = (-2x^2 + x + 2) + (-x^2 + 4x - 1)$$

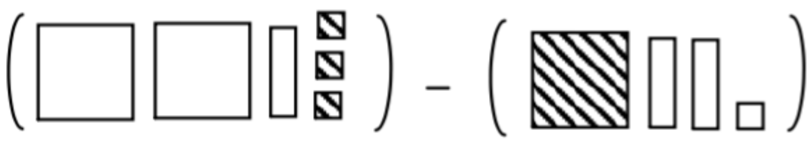



$$= -3x^2 + 5x + 1$$

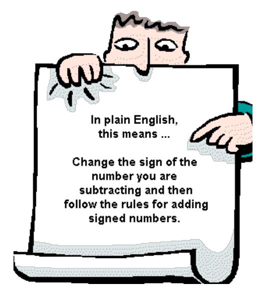
**Example:**

Subtract using algebra tiles, then simplify.

a) 

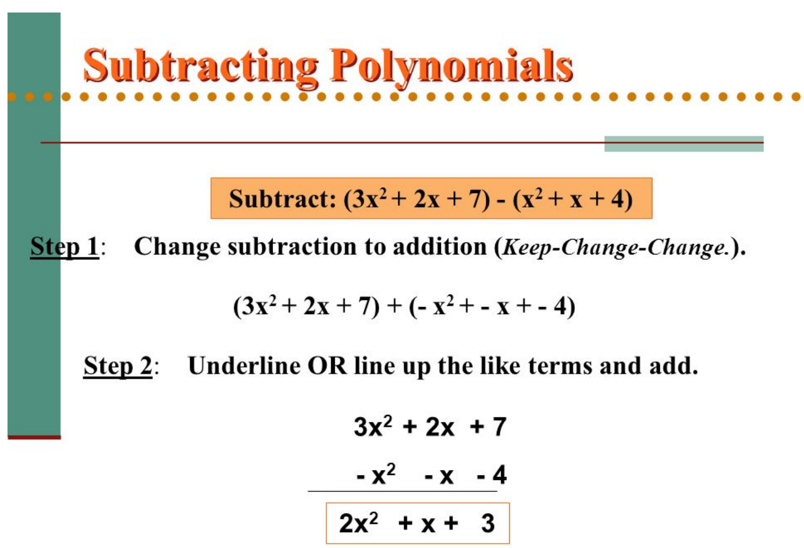
b) 

## Subtracting Polynomials Symbolically (Using Algebra)




To subtract polynomials symbolically, we need to remember to **ADD THE OPPOSITE of every term** in the second polynomial, then combine like terms.

Just like adding, we can subtract polynomials horizontally or vertically.



**Example:**

Subtract using algebra, horizontally.

a) $(3x^2 - 6x + 4) - (7x^2 + 3x - 2)$

b) $(-2a^2 + a - 1) - (a^2 - 3a + 2)$

**Example:**

Subtract using algebra, vertically.

a) $(3x^2 + 4x - 1) - (2x^2 - 3x + 2)$

b) $(-5a^2 - 3ab + 2b^2) - (8a^2 - 7ab - 4b^2)$

**Example:**

On a test a student completed a subtraction question as shown below.

$$
\begin{aligned}
&(2y^2 - 3y + 5) - (y^2 + 5y - 2) \\
&= 2y^2 - 3y + 5 - y^2 + 5y - 2 \\
&= 2y^2 - y^2 - 3y + 5y + 5 - 2 \\
&= y^2 + 2y + 3
\end{aligned}
$$

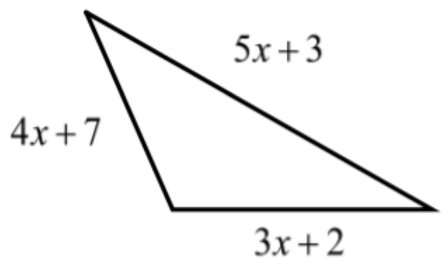
a) Explain why this solution is incorrect.

b) What is the correct answer? Show your work.

## Applications of Adding & Subtracting Polynomials

**Example 1:**

a) Write a simplified expression for the perimeter of the triangle.




b) If the value of $x = 4$ cm, what is the perimeter of the triangle?

**Example 2:**

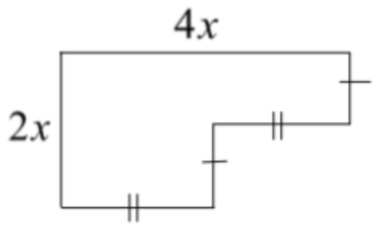
Subtract $2x^2 + 2x + 5$ **from** $5x^2 - 7x + 4$.

**Example 3:**

Subtract the sum of $(a + b)$ and $(2a - b)$ **from** $(4a - 4b)$.

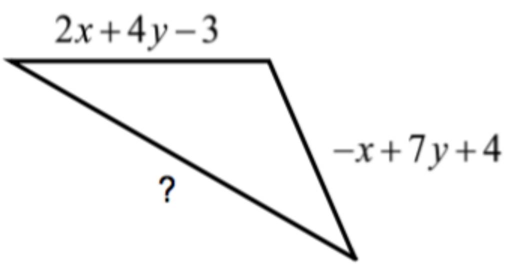
**Example 4:**

Write a monomial that describes the perimeter of the following shape.



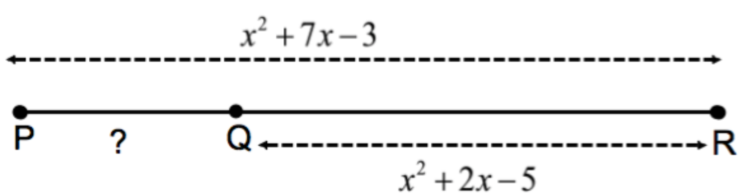

**Example 5:**

Find the missing side if the perimeter of the following triangle is $5x + 3y - 2$.



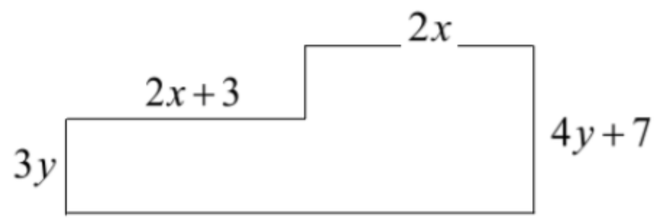

**Example 6:**

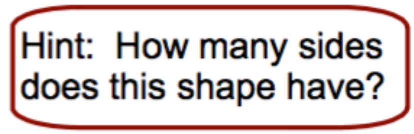
Find the length of PQ.



**Example 7:**

a) Write a simplified expression for the perimeter of the following shape.




b) What is the perimeter if $x = 1$ cm and $y = 2$ cm?

## Section 5.5 – Multiplying & Dividing a Polynomial by a Constant

Remember the signs:

When multiplying or dividing …

+ x + = +    - x + = -
- x - = +    + x - = -

+ ÷ + = +    - ÷ + = -
- ÷ - = +    + ÷ - = -

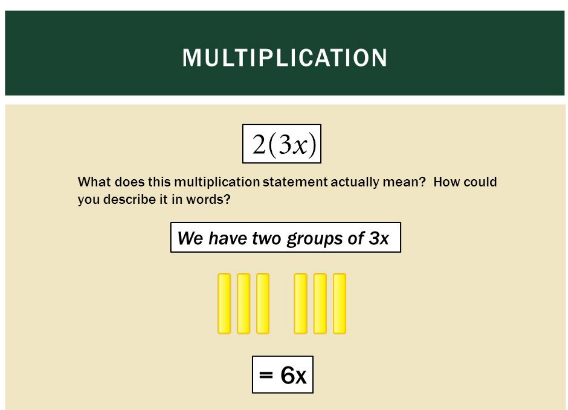
We will be expected to multiply and divide polynomials using one of the following methods:

- using algebra tiles
- using area model
- symbolically (using algebra)

### Multiplying a Polynomial by a Constant Using Algebra Tiles

To multiply a polynomial by a constant using algebra tiles we need to draw the number of sets (the constant term) of the polynomial with algebra tiles.

For example, to determine the product of $2(3x)$ using tiles, we can model the product as 2 rows of three $x$ tiles.




**Note:** To determine the product using a negative constant the process is slightly different.

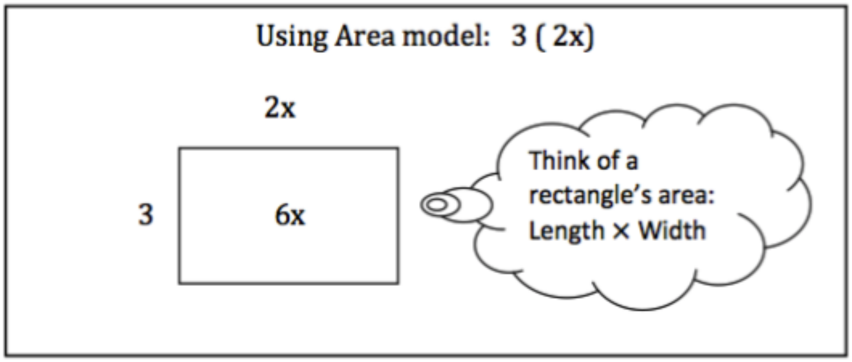
For example, to determine the product of $-2(3x)$ using tiles, we cannot model the product as $-2$ rows of three $x$ tiles. So we must change all of the the signs in order to have a positive number of rows.

In this case, $-2(3x)$ would become $2(-3x)$.

## Multiplying a Polynomial by a Constant Using Area Model

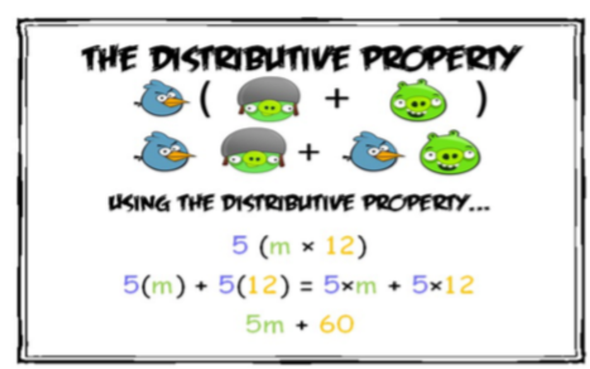
To multiply a polynomial by a constant using an area model we need to draw a rectangle with one side length as the constant and the other as the polynomial.

For example, to determine the product of $3(2x)$ using an area model, we can model the product as the area of a rectangle with side lengths 3 and $2x$.

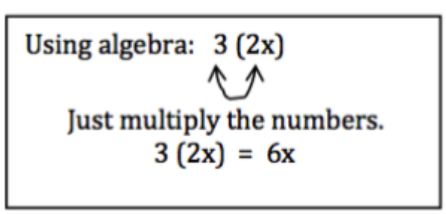



## Multiplying a Polynomial by a Constant Symbolically (Using Algebra)

To multiply a polynomial by a constant symbolically, we have to use the **distributive property**. This means that we have to multiply each term in the brackets by the term outside the brackets.




For example, to determine the product of $3(2x)$ symbolically, we just multiply the the terms together.



**Example:**

Multiply each of the following using algebra tiles, an area model and symbolically.

a) $3(2x + 2)$

Use Algebra Tiles:

Use an Area Model:

Use Algebra:

Multiply each term of the polynomial inside the bracket by the monomial in front of the bracket.

b) $-4(x + 2)$

Use Algebra Tiles:

Use an Area Model:

Use Algebra:

Multiply each term of the polynomial inside the bracket by the monomial in front of the bracket.

**Example:**

How would you sketch the following products with algebra tiles?

a) $-3(2m-4)$

b) $-(2x-1)$

**Example:**

Multiply using the distributive property … using algebra. Be careful with signs!

a) $3(-2m+4)$

b) $-8(x-5)$

c) $-2(-n^2+2n-1)$

d) $3(-8-7x)$

**Example:**

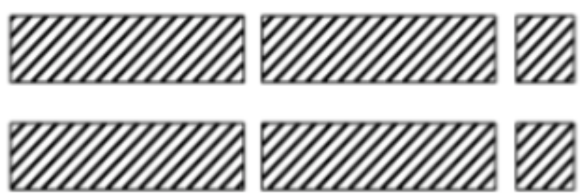
Sketch the answer to the following using an area model.

$$-2(-n^2 + 2n - 1)$$

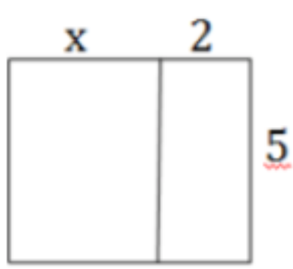
**Example:**

Determine the length, width and area of each of the following.
Write a multiplication sentence.

a)



b)



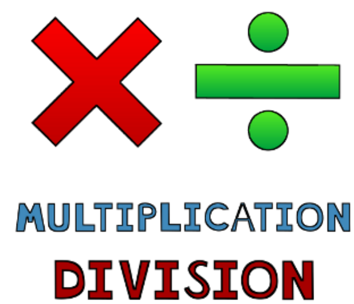
**Example:**

Determine the area of each of the following. Write a multiplication sentence.

a) Length = $2x + 2$
   Width = 4

b) Length = $-x + 3$
   Width = 2

Multiplication and division are inverse operations. So to divide a polynomial by a constant, we have to **reverse** the process of multiplication.




## Dividing a Polynomial by a Constant Using Algebra Tiles

To divide a polynomial by a constant using algebra tiles we need to arrange the polynomial into equal rows (the constant) with algebra tiles.

> When dividing by a constant term ⟶ think about arranging the tiles into equal groups.

**Example:**

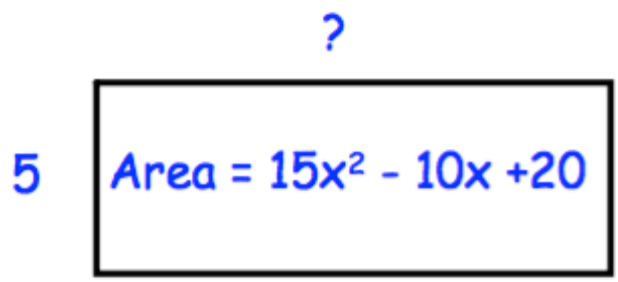
Divide each of the following using algebra tiles.

a) $\frac{4x^2}{2}$

b) $\frac{4x^2-8x}{4}$

## Dividing a Polynomial by a Constant Using Area Model

To divide a polynomial by a constant using an area model we need to draw a rectangle with one side length as the constant and the area as the polynomial.




**Example:**

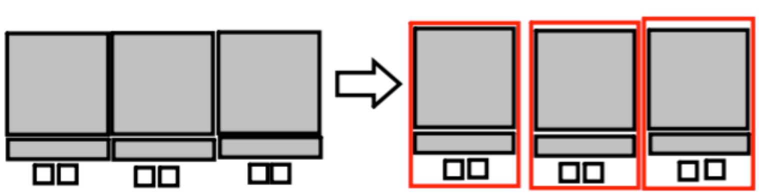
If the area of a rectangle is $21y^2 + 14y - 7$ and the width is 7 cm, what is the length?

**Example:**

The perimeter of a square is $16x^2 - 12y$. What is the length of each side of the square?

**Example:**

Write a division sentence to represent the diagram below.



## Dividing a Polynomial by a Constant Symbolically (Using Algebra)

To divide a polynomial by a constant symbolically we need to divide each term in the polynomial by the constant term.

**NOTE:**
However many terms are in the numerator, that's how many terms are in your answer. When dividing a trinomial by a monomial, you will have a trinomial answer.

$$\frac{12m^2 + 6m - 9}{3} = \frac{12m^2}{3} + \frac{6m}{3} - \frac{9}{3} = 4m^2 + 2m - 3$$

Be careful when dividing by negatives!

$$\frac{-3y^2 + 15xy - 21x^2}{-3} = \frac{-3y^2}{-3} + \frac{15xy}{-3} - \frac{21x^2}{-3} = y^2 - 5xy + 7x^2$$

**Example:**

Divide each of the following symbolically.

a). $\frac{4x^2}{2}$

Divide the numbers.
Reduce to lowest terms if possible.

b). $\frac{4x^2 - 8x}{2}$

Rewrite as two fractions.
Divide both monomials.

**Example:**

Divide each of the following using algebra.

(a) $\dfrac{12m^2+6m-9}{3}$

(b) $\dfrac{-3y^2+15xy-21x^2}{-3}$

(c) $\dfrac{24m^2}{8}$

(d) $\dfrac{-6p^2+9p-3}{-3}$

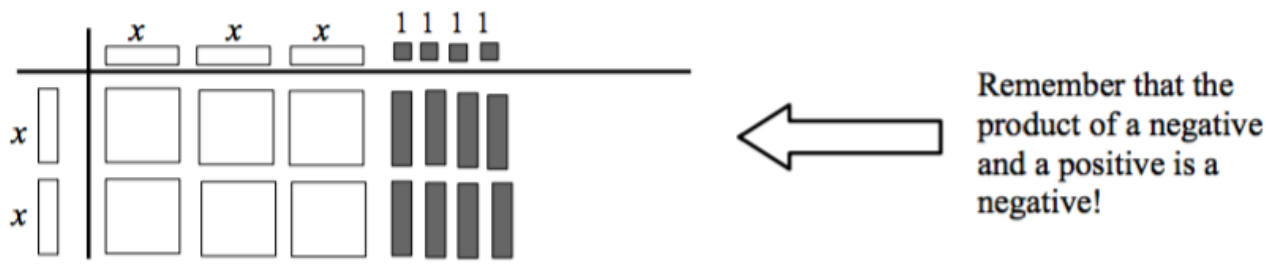
## Section 5.6 – Multiplying & Dividing a Polynomial by a Monomial

We can use the same strategies we know for multiplying and dividing a polynomial by a constant to multiply and divide a polynomial by a monomial.

### Multiplying a Polynomial by a Monomial Using Algebra Tiles

To multiply a polynomial by a monomial using algebra tiles we have to form a rectangle with **guiding tiles**. To help build the rectangle, we need to place guiding tiles to represent each dimension (these are the polynomial and the monomial). Then we fill in the rectangle with tiles.

For example, to find the product of $2x(3x - 4)$, we need to make a rectangle with dimensions $2x$ and $3x - 4$.




Six $x^2$ tiles and eight $-x$ tiles fill the rectangle, so $2x(3x - 4) = 6x^2 - 8x$

**Example:**

Multiply each of the following using algebra tiles.

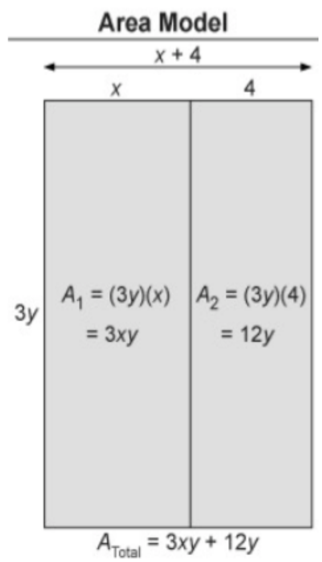
a) $x(2x + 1)$

b) $-2x(-x - 3)$

## Multiplying a Polynomial by a Monomial Using Area Model

To multiply a polynomial by a monomial using an area model we need to draw a rectangle with one side length as the monomial and the other as the polynomial.

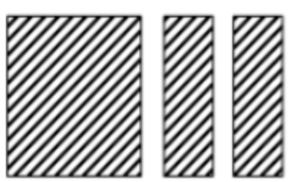
For example, to determine the product of $3y(x + 4)$ using an area model, we can model the product as the area of a rectangle with side lengths $3y$ and $x + 4$.




**Example:**

Find the area of each rectangle.

a)



b) Length = $2x$
Width = $3x$

## Multiplying a Polynomial by a Monomial Symbolically (Using Algebra)

To multiply a polynomial by a monomial symbolically, we have to use the **distributive property**. This means that we have to multiply each term in the brackets by the term outside the brackets.

For example, to find the product of $3x^2(x+5)$ we need to use the distributive property as follows:

$$3x^2(x+5)$$

$$\begin{aligned} 3x^2(x+5) &= 3x^2(x) + 3x^2(5) \\ &= 3x^2x^1 + 3 \bullet 5x^2 \\ &= 3x^3 + 15x^2 \end{aligned}$$

Remember the exponent rule: $a^n \times a^m = a^{n+m}$

**Example:**

Multiply each of the following using the distributive property.

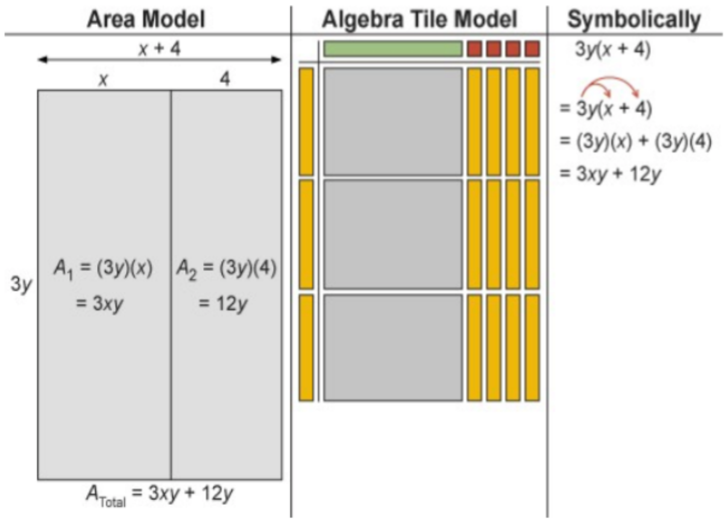
a) $5y(y+1)$

b) $6x(12-x)$

c) $-4x(2x-3)$

d) $-6m(m+4)$

Let's put all the ways together!!!!




**Example:**

Multiply $3x(2x + 2)$ using algebra tiles, an area model and symbolically.

Use Algebra Tiles

Use an Area Model

Use Algebra

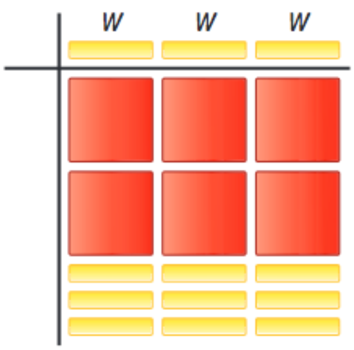
Remember the exponent rule:
$a^n \times a^m = a^{n+m}$

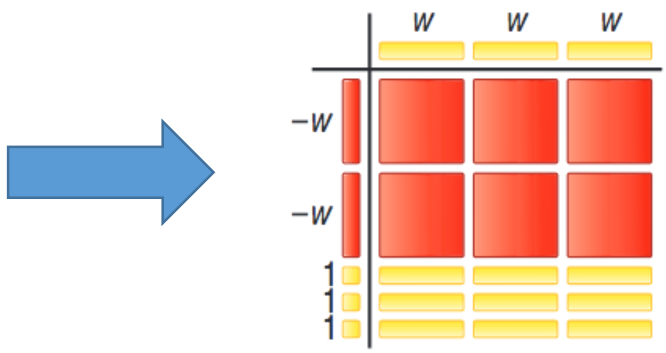
To divide a polynomial by a monomial, we have to **reverse** the process of multiplication.

**Dividing a Polynomial by a Monomial Using Algebra Tiles**

To divide a polynomial by a monomial using algebra tiles we need to arrange the polynomial into a rectangle with one side as the monomial with algebra tiles.

For example, to find the quotient of $\frac{-6w^2+9w}{3w}$ we need to arrange six $-w^2$ tiles and nine $w$ tiles in a rectangle with one side as $3w$ then fill in the missing side.




**Example:**

Divide the following using algebra tiles.

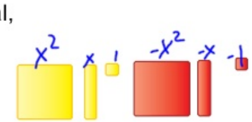
a) $\frac{4x^2}{2x}$

b) $\frac{4x^2-8x}{2x}$

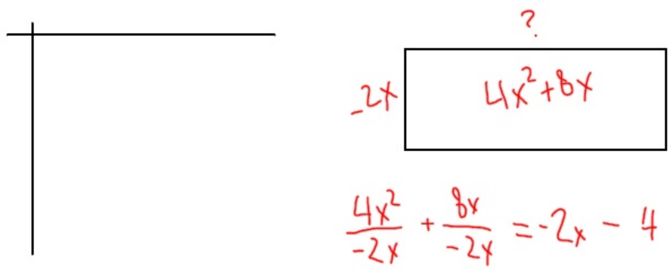
## Dividing a Polynomial by a Monomial Using Area Model

> When dividing by a term containing a variable ⟶ think about the denominator as being the width of the tiles, the numerator is the area and now find the missing length.

Model division of monomial by a monomial, or binomial by a monomial using algebra tiles or an area model.




Model $(4x^2 + 8x) \div -2x$ using both an algebra model and an area model




**Example:**

Divide each of the following using the area model.

a) $\frac{4x^2}{2x}$

b) $\frac{4x^2 - 8x}{2x}$

## Dividing a Polynomial by a Monomial Symbolically (Using Algebra)

To divide a polynomial by a monomial symbolically we have to divide each term in the polynomial by the monomial.

$$\frac{18x^4-10x^2+6x^7}{2x^2}=\frac{18x^4}{2x^2}-\frac{10x^2}{2x^2}+\frac{6x^7}{2x^2}$$

Now, we just reduce each term!

$$=9x^2-5+3x^5$$

**Example:**

Divide each of the following symbolically.

a). $\frac{4x^2}{2x}$

Divide the numbers.
For the variable,
remember the exponent rule for dividing.

b). $\frac{4x^2-8x}{2x}$

Rewrite as two fractions. Divide both monomials. Don't forget the exponent rule with the variables.

NOTE:
However many terms are in the numerator, that's how many terms are in your answer. When dividing a trinomial by a monomial, you will have a trinomial answer.

**Example:**

Simplify each of the following.

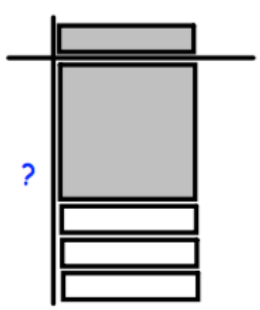
(a) $\dfrac{-15xy}{3x}$

(b) $\dfrac{-8y^2+24y}{4y}$

(c) $\dfrac{30x^2-18xy}{-6x}$

**Example:**

Divide symbolically. What is the division sentence?



**Example:**

Identify the error(s) in the solution below and then write the correct solution.

$$(12x^2 - 4x) \div (-2x)$$
$$= \frac{12x^2}{-2x} - \frac{4x}{-2x}$$
$$= -6x - 2$$
$$= -8x$$

**Example:**

Draw a rectangle with an area of $36a^2 + 12a$. How many different dimensions are possible for this rectangle?