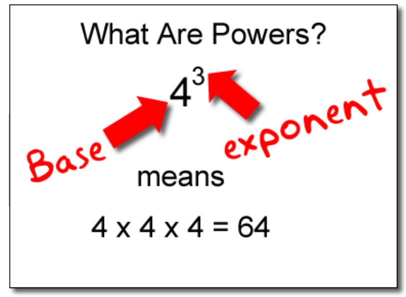
Section 2.1 – What is a Power?

When an integer, other than 0, can be written as a product of equal factors, we can write the integer as a power.



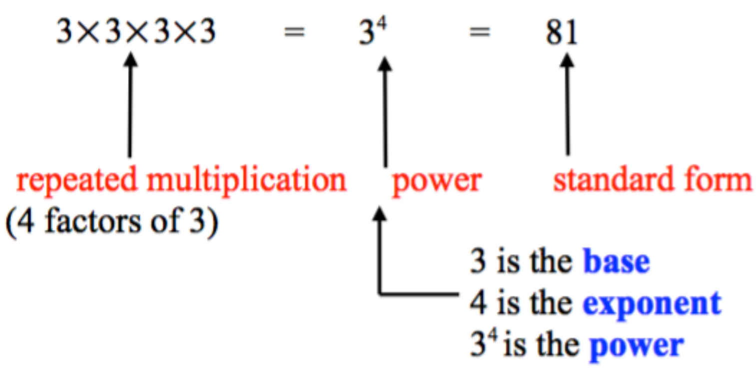
Terminology

The expression 3^4 is called a **power**.

3 is called the ______________________ .

4 is called the ______________________ .

The **exponent** tells us how many times the **base** gets multiplied.



Question:

Are the base and exponent interchangeable? In other words, does $2^5 = 5^2$?

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ $\qquad$ $5^2 = 5 \times 5 = 25$

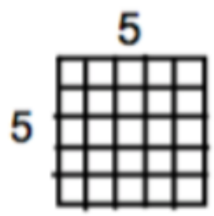
No! The base and the exponent cannot be switched and still be equal.

Challenge:

Can you think of one example where the base and exponent can be switched and the answers are still equal? You **cannot** use the same number for both!

A power with an integer base and an exponent of 2 is a **square number**.

When the base is a positive integer, we can illustrate a square number using an **area model**.



There are three ways to write 25:

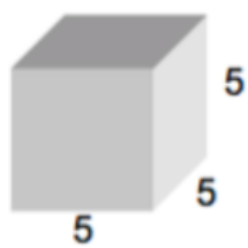
Standard Form: ____________________

Repeated Multiplication: ____________________

Power: ____________________

A power with an integer base and an exponent of 3 is a **cube number**.

When the base is a positive integer, we can illustrate a cube number using a **volume model**.



There are three ways to write 125:

Standard Form: ____________________

Repeated Multiplication: ____________________

Power: ____________________

Example:

Write as a power.

a) $3\times3\times3\times3$

b) $6\cdot6\cdot6\cdot6\cdot6\cdot6$

c) $(-2)(-2)(-2)$

d) $(1)(1)(1)(1)(1)(1)$

e) 12

f) $-(3)(3)$

Example:

Write as repeated multiplication.

a) 2^7

b) $(-3)^2$

Example:

Write in standard form.

a) 2^4

b) $(-5)(-5)(-5)(-5)$

Note: You have to be careful when evaluating expressions with negative signs!!!

$(-3)^2 = -3 \times -3 = 9$ The base in this power is -3

$-3^2 = -(3)^2 = -(3 \times 3) = -9$ The base in this power is just 3 (**not** -3) **Why?**

$-(-3)^2 = -(-3 \times -3) = -9$ The base in this power is -3

The Importance of Brackets

$(-3)^2$ The brackets tell us that the base is −3.

- $(-3)^2 = (-3) \times (-3) = +9$

When there is an EVEN NUMBER of negatives then the product is POSITIVE.

- $(-3)^3 = (-3) \times (-3) \times (-3) = -9$

When there is an ODD NUMBER of negatives then the product is NEGATIVE.

-3^2 There are no brackets so the base is 3. The negative applies to the whole expression.

- $-3^2 = -(3 \times 3) = -9$

Example:

Evaluate. Identify the base.

a) $(-2)^4$ base: _______ b) -2^4 base: _______

c) $-(-2)^4$ base: _______ d) $-(-2^4)$ base: _______

Section 2.2 – Powers of Ten & the Zero Exponent?

Powers of Ten

Power	Expression	Standard Form
10^1	10	10
10^2	10 x 10	100
10^3	10 x 10 x 10	1,000
10^4	10 x 10 x 10 x 10	10,000
10^5	10 x 10 x 10 x 10 x 10	100,000
10^6	10 x 10 x 10 x 10 x 10 x 10	1,000,000
10^7	10 x 10 x 10 x 10 x 10 x 10 x 10	10,000,000
10^8	10 x 10 x 10 x 10 x 10 x 10 x 10 x 10	100,000,000
10^9	10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10	1,000,000,000

A **power of 10** is a power with a base of 10 and an integer exponent.

The exponent is equal to the number of zeroes when the power is written in standard form.

$10^1 = 10$

$10^2 = 10 \times 10 = 100$

$10^3 = 10 \times 10 \times 10 = 1,000$

$10^4 = 10 \times 10 \times 10 \times 10 = 10,000$

$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100,000$

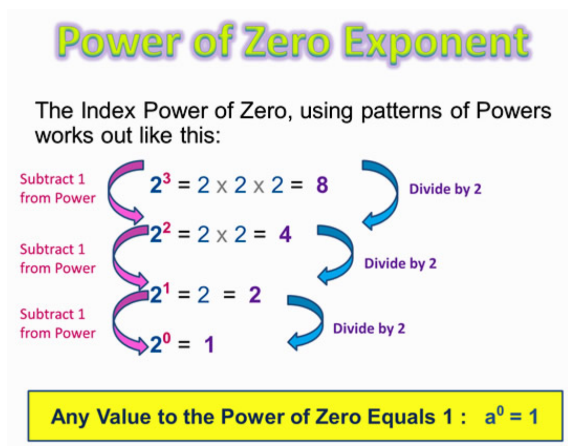
$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000$

Following this pattern, what should 10^0 be?

The exponent is 0 so there should be no zeroes!!! So $10^0 = 1$

This leads us to the **Zero Exponent Law**.

The Zero Exponent Law states: *"**that a power with an integer base, other than 0, and an exponent of 0 is equal to 1.**"*



$2^0 = 1$ $13^0 = 1$ $(-5)^0 = 1$ $100^0 = 1$ $5345^0 = 1$ $(2 + 3)^0 = 1$

But, $-5^0 = -1$ Why?

Example:

Evaluate. Watch your order of operations!

a) $(560)^0$

b) $(2\times4)^0$

c) $3+2^0$

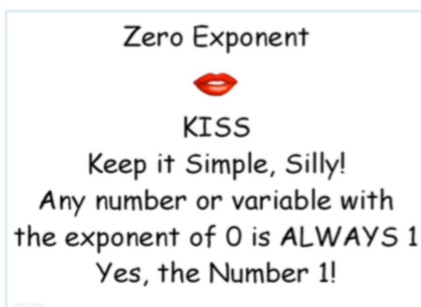
d) 3^0+2^0

e) $(3+4\times2)^0$

f) $-(3+2)^0$

g) -3^0+5

h) $-3^0+(-2)^0$



Example:

Write each of the following as a power of 10.

a) 10

b) 100

c) 1000

d) 10000

e) one million

f) ten billion

We can use the zero exponent and the powers of 10 to write a number in expanded form with powers

The number 4856 in expanded form is:

Method a)

$4000 + 800 + 50 + 6$

Method b)

$4 \times 1000 + 8 \times 100 + 5 \times 10 + 6 \times 1$

Method c)

$4 \times 10^3 + 8 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$

Example:

Write each of the following using powers of 10.

a) 123

b) 1045

c) 12340

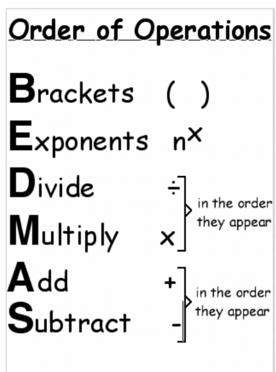
d) 305068

Section 2.3 – Order of Operations with Powers

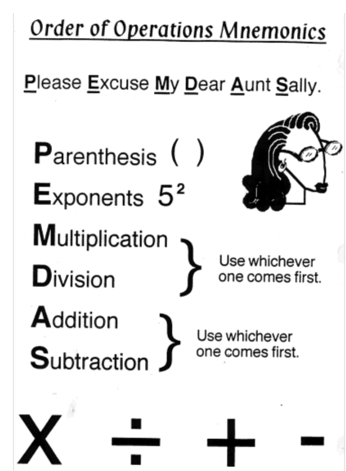
To avoid getting different answers when we evaluate an expression, we must use this order of operations:

- do the operations in the **brackets**
- evaluate the **exponents**
- **multiply** and **divide**, in order from left to right
- **add** and **subtract**, in order from left to right

We can use the word **BEDMAS** to help us remember the order of operations.



Or we could use any other way that would assist us in remembering the order. For example, we could also use:



Example:

Evaluate.

a) $2^3 + 1$

b) $8 - 3^2$

c) $(3 - 1)^3$

d) $[2 \times (-2)^3]^2$

e) $(7^2 + 5^0) \div (-5)^1$

f) $\frac{4+2^4}{2^2}$

Example:

This student got the correct answer, but did not earn full marks. Find and explain the mistake the student made.

$$-(24 - 3 \times 4^2)^0 \div (-2)^3$$
$$-(24 - 12^2)^0 \div (-8)$$
$$-(24 - 144)^0 \div (-8)$$
$$-(-120)^0 \div (-8)$$
$$-1 \div (-8)$$
$$\frac{1}{8}$$

Section 2.4 – Exponent Laws I

Patterns arise when we **multiply and divide powers with the same base**.

Multiplication

$3^5 \times 3^3 = (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3)$

$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ ***since it's all multiplication we don't need brackets**

$= 3^8$

$7^2 \times 7^4 = (7 \times 7) \times (7 \times 7 \times 7 \times 7)$

$= 7 \times 7 \times 7 \times 7 \times 7 \times 7$

$= 7^6$

What do we notice?

- *"When we multiply powers with the **same base**, the answer has the same base and the exponent is the **sum of the exponents** of the original powers."*

Exponent Law 1

"To multiply powers with the same base, add the exponents."

$$a^n \cdot a^m = a^{n+m}$$

Example:

Write as a single power first, then evaluate.

a) $4^3 \times 4^4$

b) $7^5 \times 7^{-5}$

c) $(-3)^2 \times (-3)^4$

Example:

Write as a single power.

a) $9^5 \times 9$

b) $8^{-11} \times 8^{13}$

c) $3.8^4 \times 3.8^2$

d) $\left(\frac{1}{4}\right)^{12} \times \left(\frac{1}{4}\right)^8$

e) $5^2 \times 5 \times 5^3$

f) $(-2)^3 \times (-2)^0 \times (-2)^{-2}$

Example:

Use the laws of exponents to simplify and then evaluate.

a) $3^4 \times 3^2 \times 3^3$

b) $7^5 \times 7^2 \times 7$

c) $(-2)^3 - (-2)^2 \times (-2)$

d) $4^2 - 3^0 + 2^3(2)^2$

Note:

- "Evaluate" means to find the answer in "standard form" (as a regular number)
- "Express as a single power" means leave your answer in exponent form

<u>Division</u>

$$3^5 \div 3^3 = \frac{3^5}{3^3}$$

$$= \frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3}$$

***divide the numerator and denominator by their common factors**

$$= 3 \times 3$$

$$= 3^2$$

What do we notice?

- *"When we divide powers with the* ***same base****, the answer has the same base and the exponent is the* ***difference of the exponents*** *of the original powers."*

<u>Exponent Law 2</u>

"To divide powers with the same base, subtract the exponents."

$$\frac{a^n}{a^m} = a^{n-m}$$

Example:

Write as a single power first, then evaluate.

a) $2^5 \div 2^2$

b) $\frac{(-6)^8}{(-6)^6}$

c) $\frac{3^4}{3^4}$

Example:

Write as a single power.

a) $12^6 \div 12$

b) $\frac{8^3}{8^{-2}}$

c) $1.4^6 \div 1.4^2$

d) $\frac{x^7}{x^5}$

e) $\frac{5^7}{5^3}$

f) $(-2)^4 \div (-2)^0$

Example:

Use the laws of exponents to simplify and then evaluate.

a) $6^{-6} \div 6^4$

b) $8^{12} \div 8^7 \times 8^2$

c) $\frac{(-4)^{10}}{(-4)^3 \times (-4)^3}$

d) $6^2 + 6^3 \times 6^2$

Example:

Write as a single power where possible, then evaluate.

a) $5^3 \times 5^2 - 5^2 \times 5$

b) $-3^4(3^6 \div 3^3) + 3^2$

c) $-2^3 - 2^6 \div 2^4$

d) $(-3)^6 \div (-3)^5 - (-3)^2 \times (-3)$

Section 2.5 – Exponent Laws II

We can use the exponent laws from the previous section to simplify powers written in other forms.

The Laws of Exponents

Product Law
Quotient Law
Power of a Power Law
Power of a Product Law
Power of a Quotient Law
Zero Exponent Law
Negative Exponent Law

Power of a Power

We can raise one power to another power.

$(3^2)^3$ is a power of a power. It means $3^2 \times 3^2 \times 3^2$.

$3^2 \times 3^2 \times 3^2 = (3 \times 3) \times (3 \times 3) \times (3 \times 3)$

$= 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 3^6$

What do we notice?

- *"When we raise a power to another power, the answer has the same base as the original power and the exponent is the* ***product of the exponents****."*

Exponent Law 3

"To raise a power to a power, multiply the exponents"

$$(a^n)^m = a^{n \cdot m}$$

Example:

Write as a power.

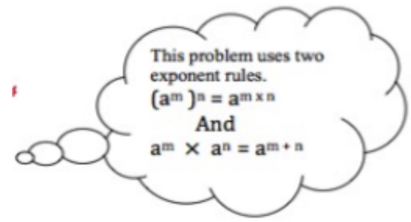
a) $(3^2)^4$

b) $[(-7)^3]^2$

c) $-(2^2)^4$

d) $(3^0)^2$

e) $(42^3)^2 \times (42^4)^4$



Example:

Simplify first, then evaluate.

a) $(2^3)^2 \times (3^2)^2$

b) $(-3^2)^3 \times (-3^0)^9$

c) $(3^2 \times 3^8) \div (3^3)^2$

d) $(2^3 \times 2^2) - (3^2 \times 3)^2$

Power of a Product

The base of a power may be a product.

$(2 \times 3)^3$ is a power of a product. It means $(2 \times 3) \times (2 \times 3) \times (2 \times 3)$.

$(2 \times 3) \times (2 \times 3) \times (2 \times 3) = 2 \times 3 \times 2 \times 3 \times 2 \times 3$ ***remove brackets**

$= 2 \times 2 \times 2 \times 3 \times 3 \times 3$ ***group common factors**

$= 2^3 \times 3^3$

What do we notice?

- *"When we have a power of a product, the answer is written as repeated multiplication with powers. Each term of the product gets the exponent of the power."*

Exponent Law 4

$$(ab)^n = a^n b^n$$

Example:

Evaluate each question using two ways. Use power of a product and BEDMAS.

a) $[(-7) \times 5]^2$

b) $-(3 \times 2)^2$

Example:

Evaluate, using any method of your choice.

a) $(3\times4)^3$

b) $[(-2)^2\times(-2)^1]^3$

c) $(2^3\times2^2)^3-(3^2\times3)^2$

Power of a Quotient

The base of a power may be a quotient.

$\left(\frac{2}{3}\right)^3$ is a power of a quotient. It means $\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)$.

$\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)=\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}$ ***remove brackets**

$=\frac{2\times2\times2}{3\times3\times3}$ ***group common factors**

$=\frac{2^3}{3^3}$

What do we notice?

- *"When we have a power of a quotient, the answer is written as repeated multiplication with powers. Each term of the quotient gets the exponent of the power."*

Exponent Law 5

$$\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$$

Example:

Evaluate each question using two ways. Use power of a quotient and BEDMAS.

a) $[(-24)\div 6]^4$

b) $\left(\frac{52}{13}\right)^3$

Challenge:

Use +, –, ×, ÷ to complete this equation.

$$5^2 \;\_\_\_\; 16 \;\_\_\_\; 2^2 \;\_\_\_\; 6 = 83$$