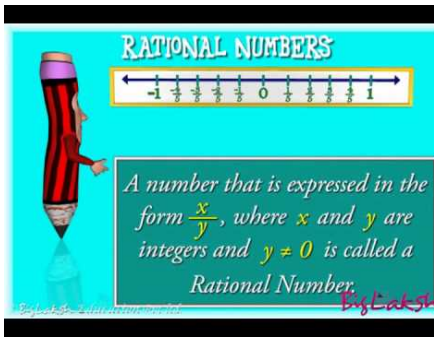
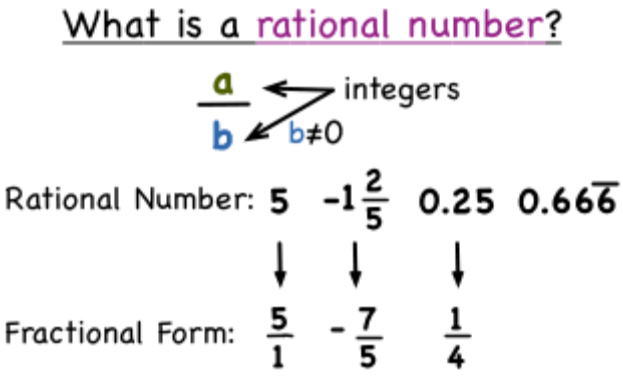
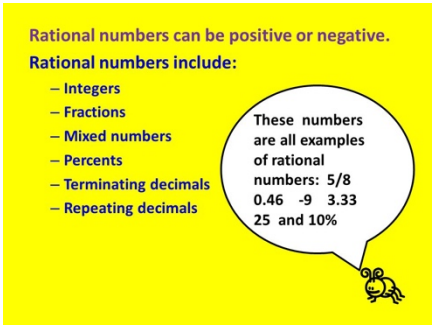
Section 3.1 – What is a Rational Number?

A **rational number** is any number that can be written as a fraction with an integer numerator and a non-zero integer denominator.

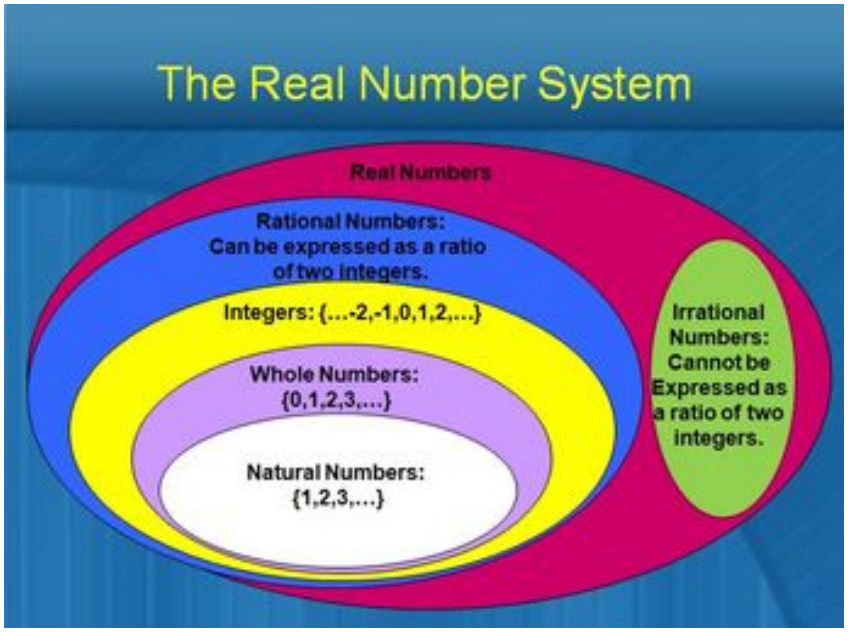




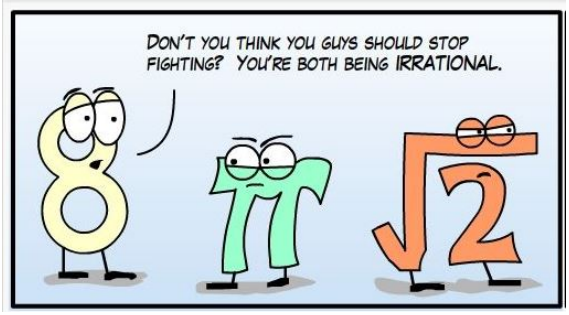


Rational numbers include:

- whole numbers
- integers
- positive and negative fractions
- repeating and terminating decimals
- percents



Not all numbers can be written as fractions. For example, π and $\sqrt{2}$ are numbers that change into non-repeating, non-terminating decimals. These are called **irrational numbers**.



IRRATIONAL NUMBERS

Numbers that CANNOT be represented as a simple fraction

Important Property	Examples
Decimals NEVER REPEAT	$\pi = 3.1415926\ldots$
Decimals NEVER END	$\sqrt{2}$

Recall,
To convert between decimals and fractions we need to do the following:

Decimals to Fractions

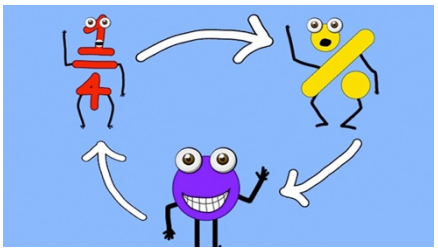
Decimal ⟶ Fraction

$$.15 = \frac{15}{100} \overset{\div 5}{\underset{\div 5}{=}} \frac{3}{20}$$

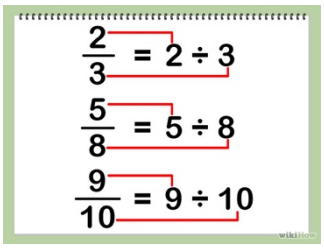
The last digit is in the hundredths place.

Use the place value of the last digit to write as fraction with denominator of 10, 100, 1000 etc. Then simplify the fraction if possible.

Fractions to Decimals



The fraction bar represents division. So to change a fraction into a decimal we need to divide the numerator (top #) by the denominator (bottom #).



Example:

Identify the rational numbers below.

a) $-\frac{1}{4}$ b) $\sqrt{9}$ c) $\frac{-4}{-9}$ d) $\sqrt{75}$ e) π f) 2.5

Example:

Convert the following decimals into fractions.

a) 0.5

b) 0.03

c) 2.5

d) -0.12

Example:

Convert the following fractions into decimals.

a) $\frac{3}{4}$

b) $\frac{2}{7}$

c) $\frac{-6}{3}$

d) $\frac{7}{8}$

Example:

Compare the following.

a) $\frac{-6}{3}$ b) $-\frac{6}{3}$ c) $\frac{6}{-3}$

What do you notice?

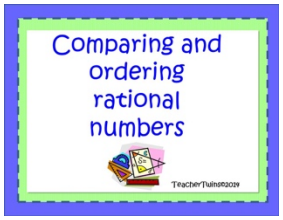
They all have the same solution!

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

but remember that:

$$\frac{-a}{-b} = -\left(\frac{-a}{b}\right) = -\left(\frac{a}{-b}\right) = \frac{a}{b}$$

Compare and Order Rational Numbers



Example:

Use < , > , or = to compare these rational numbers. Show workings!

a) $\frac{4}{7} \square \frac{5}{9}$ b) $-\frac{3}{8} \square -\frac{5}{8}$

Use common denominators then compare numerators!

c) $\frac{2}{7} \square \frac{2}{9}$ d) $\frac{-2}{7} \square \frac{-2}{9}$

e) $\frac{-3}{4} \square \frac{3}{4}$ f) $\frac{-10}{4} \square -2.8$

g) $\frac{-7}{8} \square \frac{7}{-8}$

Remember:

If the question contains only fractions - work in fractions.

If the question contains only decimals - work in decimals.

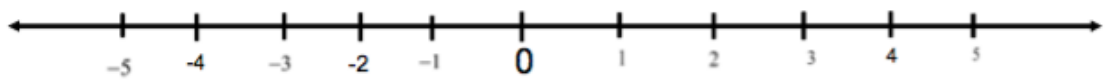
If the question contains both fractions AND decimals - your choice!

Example:

Place these rational numbers in descending order. The number line may help you.

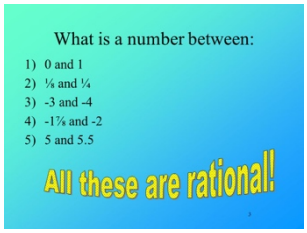
$$\frac{-3}{4},\ 0.5,\ -1.8,\ -5,\ \frac{7}{3},\ 2,\ -3.\overline{3},\ 1\frac{3}{4}$$

Since the question contains both fractions and decimals you can use either fractions or decimals to compare!



Descending order (from largest to smallest):

Writing a Rational Number Between Two Given Numbers



Example:

Identify a decimal between each pair of rational numbers.

a) $\frac{-1}{2}$ and $\frac{-1}{4}$

b) -0.25 and -0.26

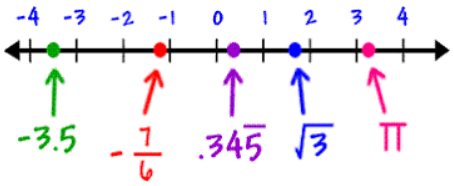
Example:

Identify a fraction between each pair of rational numbers.

a) $\frac{-2}{3}$ and $\frac{-3}{4}$

b) $\frac{5}{2}$ and $\frac{7}{3}$

Placing Rational Numbers on a Number Line



Representation of Rational Numbers on the Number Line

To express rational numbers appropriately on the number line, divide each unit length into as many number of equal parts as the denominator of the rational number and then mark the given number on the number line.

Example:

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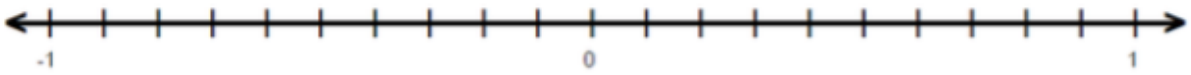
Place these fractions in order using the number line.

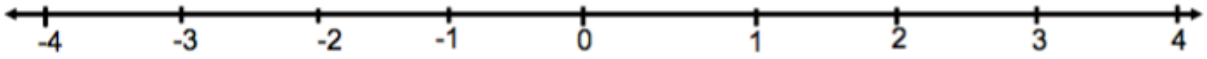
$$\frac{1}{2}, \frac{-3}{5}, \frac{1}{10}, \frac{2}{5}, \frac{-7}{10}$$



Example:

Place these decimals in order using the number line.

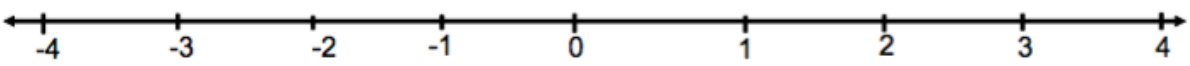
$$3.2, -1.3, 0.1, -2.7, 2.1$$



Example:

Place these rational numbers in order using the number line.

$$\frac{2}{7}, -1.3, 2\frac{5}{6}, -2\frac{3}{4}, 1.8$$



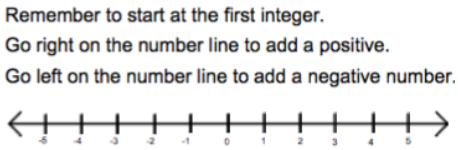
Section 3.2 – Adding Rational Numbers

To add rational numbers, we have to follow the rules for adding integers and fractions.

Integer Rules for Adding

- To add a positive integer we move to the right (go up)
- To add a negative integer we move to the left (go down)

Remember that the first number is our starting position.



Example:

Add the following.

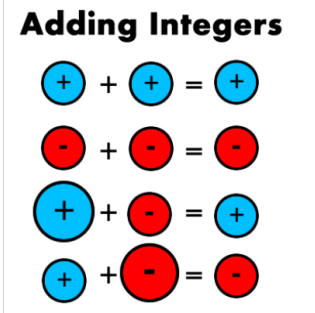
a) $(-1)+(+2)$ b) $(-7)+(+4)$ c) $-2+(-6)$

d) $(-2)+(-1)$ e) $(-6)+(-4)$ f) $(+8)+(-12)$

g) $(+5)+(-19)$ h) $(-5)+3+(-9)$ i) $7+(-2)+(-7)+(+4)$

Question:

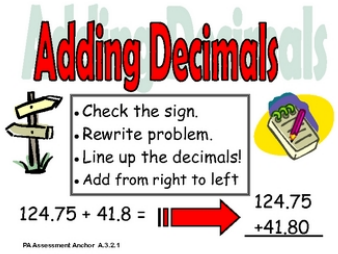
Is there a way to determine whether or not the answers to these sums will positive or negative without using a number line?



If the signs are the same: _____

If the signs are different: _____

Decimal Rules for Adding



We follow the same rules as the rules for integers.

Example:

Add the following.

a) $(-1.3) + (2.1)$

b) $(+1.9) + (1.2)$

c) $(-2.8) + (-6.5)$

d) $(-7.3) + (3.1)$

e) $(2.4) + (-1.7)$

f) $(-3.5) + 6.3$

g) $(-4.1) + (-3.1)$

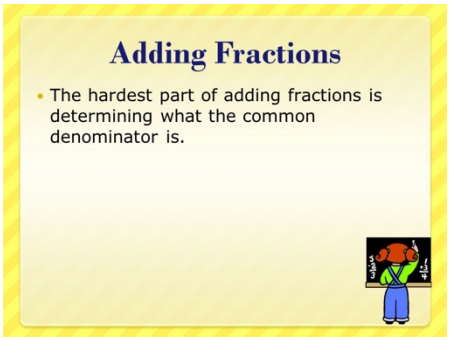
h) $(0.67) + (-0.83)$

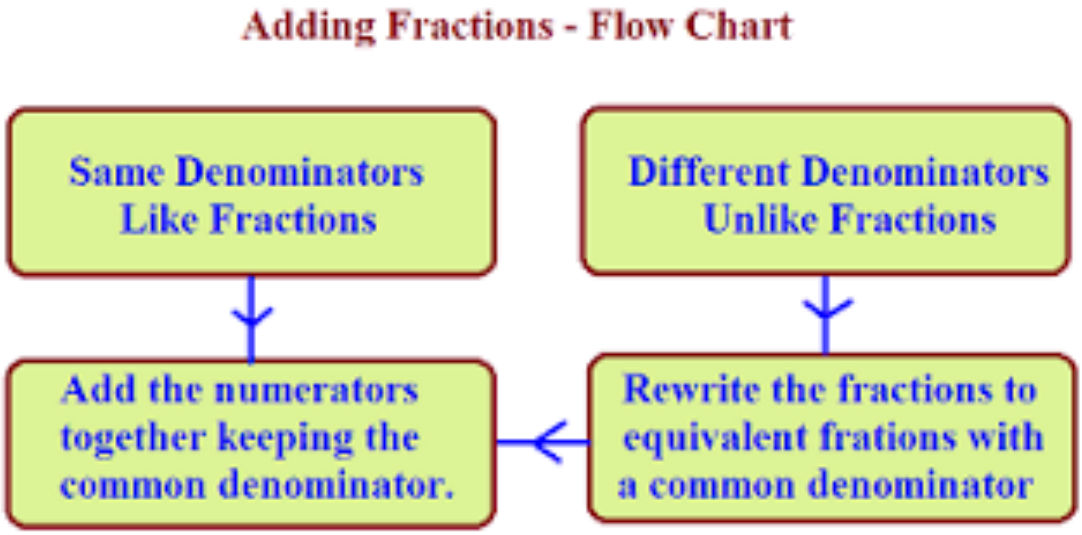
i) $-1.5 + 1.25$

k) $-0.583 + 0.625$

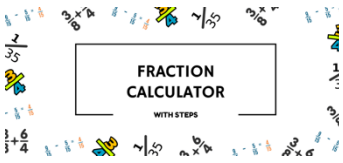
Fraction Rules for Adding

- To add fractions we need a common denominator.
- Once we have a common denominator, we add the numerators only (using the integer rules), leaving the denominator the same.
- Reduce to lowest terms, if possible.





Note: You must show all workings in order to receive full marks!!! (Even if you are using a calculator!)



$$\frac{2}{15} + \frac{3}{5} = ?$$

$$\frac{2}{15} + \frac{3 \times 3}{5 \times 3}$$

$$\frac{2}{15} + \frac{9}{15} = \frac{2+9}{15} = \frac{11}{15}$$

Same

- fractions-math.blogspot.com

Example:

Add the following.

a) $\frac{-7}{9}+\frac{5}{9}$

b) $\frac{2}{5}+\frac{-3}{5}$

Remember to:
- get common denominators
- change to improper fractions when necessary
- write answers in lowest terms

c) $\frac{-7}{8}+\frac{3}{4}$

d) $-3\frac{1}{3}+2\frac{5}{6}$

e) $1\frac{1}{2}+\left(-2\frac{1}{3}\right)$

f) $\frac{3}{8}+\frac{7}{6}$

g) $\frac{-3}{2}+\frac{1}{6}$

Example:

Complete these word problems. Your answer must include an addition sentence.

a) A guardrail needs to be exactly 19.77 m long. A contractor has 3 pieces measuring 2.21 m, 9.14 m and 3.21 m. Does he have enough to complete the guardrail?

b) Peter estimates that it takes him $\frac{1}{4}$ hour to prepare the dough, $\frac{1}{10}$ hour to grate the cheese, $\frac{1}{3}$ hour to prepare the toppings and $\frac{2}{5}$ hour to bake the pizza. What fraction of time does it take Peter in total to prepare the pizza? How many minutes is this?

Section 3.3 – Subtracting Rational Numbers

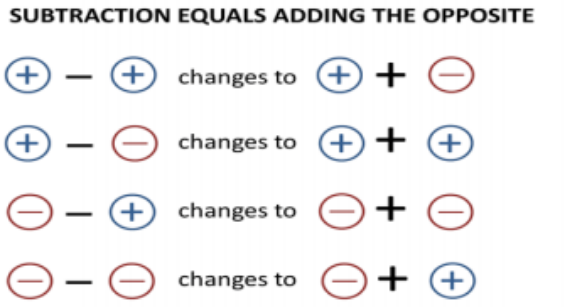
To subtract rational numbers, we have to follow the rules for subtracting integers and fractions.

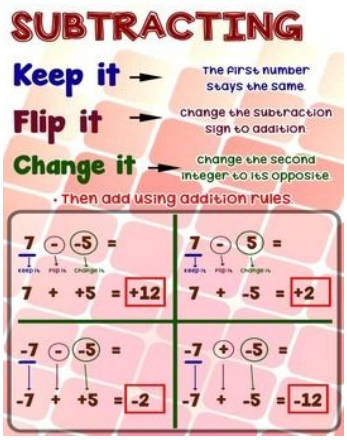
To subtract rational numbers we ADD THE OPPOSITE. Every subtraction problem can be rewritten as an addition problem.

Integer Rules for Subtracting

- To subtract a positive integer we move to the left (go down)
- To subtract a negative integer we move to the right (go up)

Remember that the first number is our starting position.





Example:

Subtract the following.

a) $(+5) - (+3)$

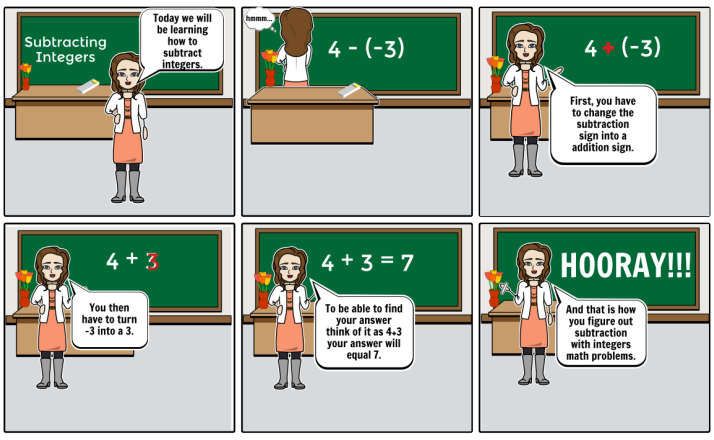
b) $7 - (-4)$

c) $(-8) - (+2)$

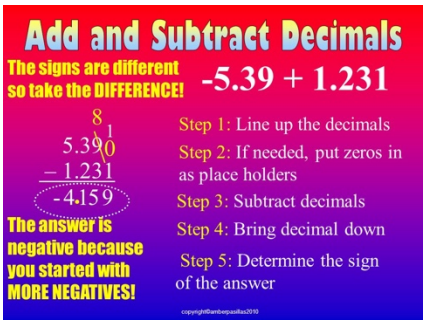
d) $(-5) - (-3)$

e) $(-4) - (-2) + 3$

f) $10 - (-3) - (-5) - 7$



Decimal Rules for Subtracting



We follow the same rules as the rules for integers.

Example:

Subtract the following.

a) $(-1.3) - (2.1)$

b) $(+1.9) - (1.2)$

c) $(-2.8) - (-6.5)$

d) $(-7.3) - (3.1)$

e) $(2.4) - (-1.7)$

f) $(-3.5) - 6.3$

g) $(-4.1) - (-3.1)$

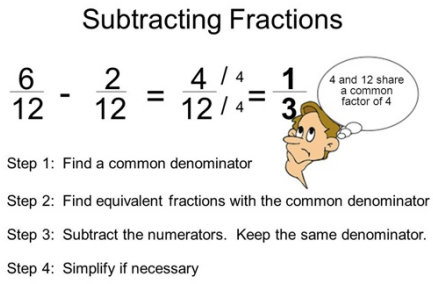
h) $(0.67) - (-0.83)$

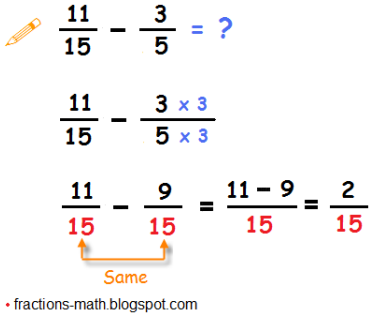
i) $-1.5 - 1.25$

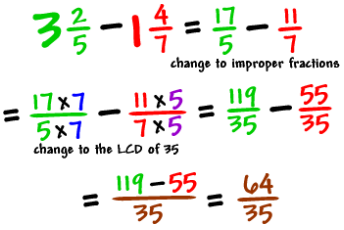
j) $-0.583 - 0.625$

Fraction Rules for Subtracting

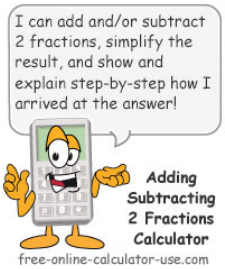
- To subtract fractions we need a common denominator.
- Once we have a common denominator, we subtract the numerators only (using the integer rules), leaving the denominator the same.
- Reduce to lowest terms, if possible.







Note: You must show all workings in order to receive full marks!!!
(Even if you are using a calculator!)



Example:

Subtract the following.

a) $\frac{5}{7}-\frac{-3}{7}$

b) $\frac{-4}{5}-\left(\frac{-1}{2}\right)$

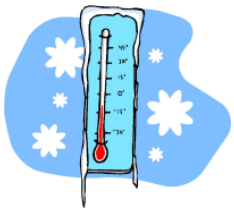
c) $-1\frac{1}{4}-\left(-2\frac{2}{3}\right)$

d) $1\frac{1}{6}-\frac{3}{4}$

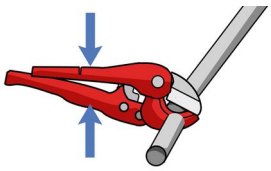
Example:

Complete these word problems. Your answer must include a subtraction sentence.

a) The temperature in St. John's is 2.5°C. In Corner Brook it is 8°C colder. What is the temperature in Corner Brook?



b) A piece of pipe is 146.3 cm long. A piece 13.7 cm is cut off. How long is the remaining piece?



c) A person climbs $12\frac{2}{3}$ meters above the water to the top of a cliff.. He dives into the water and reaches $-3\frac{1}{6}$ meters below the surface. What is the difference in these heights?

d) Which expression has the same answer as $-2.3-(-3.9)$?

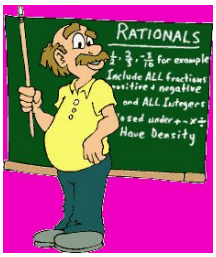
A) $-2.3+(-3.9)$ B) $2.3-(-3.9)$

C) $-2.3-(+3.9)$ D) $-2.3+(+3.9)$

e) Determine the missing number in each subtraction sentence.

$2.5-\_\_=3.8$ $\_\_-\frac{-3}{10}=\frac{2}{5}$

Section 3.4 – Multiplying Rational Numbers



To multiply rational numbers, we have to follow the rules for multiplying integers and fractions.

Integer Rules for Multiplying

Positive x Positive = POSITIVE answer
Negative x Negative = POSITIVE answer

} If BOTH integers have SAME SIGN...answer is POSITIVE

Positive x Negative = NEGATIVE answer
Negative x Positive = NEGATIVE answer

} If the integers have DIFFERENT SIGNS...answer is NEGATIVE

Example:

Multiply the following.

a) $(+5) \times (+3)$

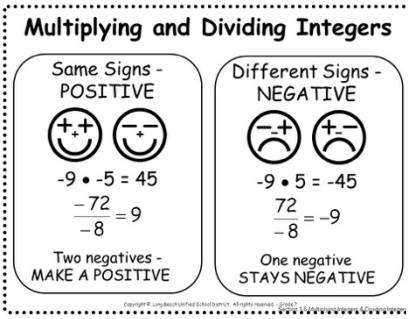
b) $7 \times (-4)$

c) $(-8) \times (+2)$

d) $(-5) \times (-3)$

e) $(-4) \times (-2) \times 3$

f) $10 \times (-3) \times (-5) \times 7$



Note: When you have more than 2 integers, you have to work with 2 integers at a time!!

You can use these rules even if you're multiplying more than two numbers together.

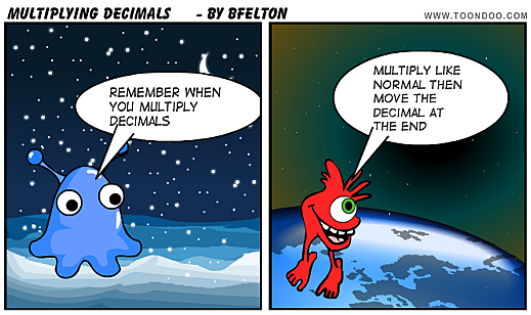
Just **count the number of "–" signs** in the question.

Rules for multiplying integers
positive × positive = positive
positive × negative = negative
negative × positive = negative
negative × negative = positive

If there's an **even** number of negative factors, they'll cancel out in pairs, and the answer will be **positive**.

If there's an **odd** number of negative factors, you'll end up with one that doesn't cancel out, so the final answer will be **negative**.

Decimal Rules for Multiplying



We follow the same rules as the rules for integers.

Example:

Multiply the following.

a) $(-1.3) \times (2.1)$

b) $(+1.9) \times (1.2)$

c) $(-2.8) \times (-6.5)$

d) $(-7.3) \times (3.1)$

e) $(2.4) \times (-1.7)$

f) $(-3.5) \times 6.3$

g) $(-4.1) \times (-3.1)$

h) $(0.67) \times (-0.83)$

i) -1.5×1.25

j) -0.583×0.625

Fraction Rules for Multiplying

- To multiply fractions we **don't** need a common denominator.
- We multiply the numerators (using the integer rules).
- We multiply the denominators (using the integer rules).
- Reduce to lowest terms, if possible.

Multiplying Fractions

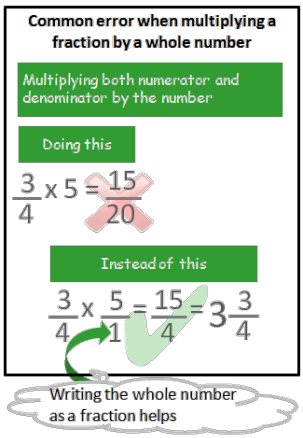
1. Multiply straight across.
2. Simplify if needed.

Example:

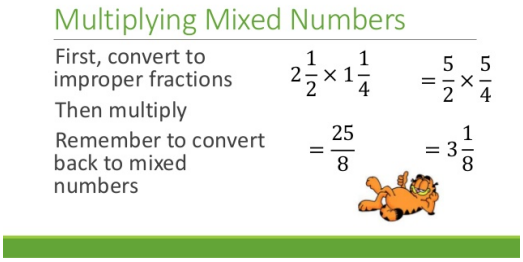
Multiply the following.

a) $\left(-\frac{2}{5}\right)\times\frac{3}{8}$

b) $3\times\frac{-5}{8}$



c) $2\frac{1}{4}\times\left(-\frac{2}{3}\right)$



Note: We can reduce after we multiply but we can also reduce **before** we multiply.

Multiplying Fractions (Option 2)

1. Simplify first.
2. Multiply straight across.

Cross reduce:

$$\frac{-11}{7} \times \frac{-21}{44}$$

Cross reduction is helpful when multiplying large numbers. It is a shortcut to use should you choose to!

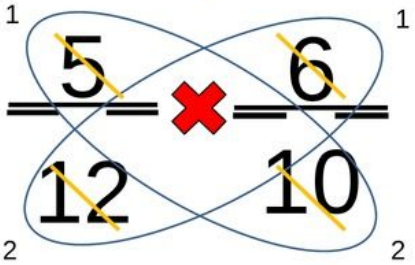
Example:

Reduce first, then multiply.

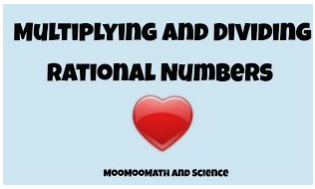
a) $\frac{8}{3} \times \frac{-7}{4}$

b) $\frac{9}{16} \times \frac{14}{3}$

Butterfly Method



Section 3.5 – Dividing Rational Numbers



To divide rational numbers, we have to follow the rules for dividing integers and fractions.

Integer Rules for Dividing

Positive ÷ Positive = POSITIVE answer
Negative ÷ Negative = POSITIVE answer

} If BOTH integers have SAME SIGN...answer is POSITIVE

Positive ÷ Negative = NEGATIVE answer
Negative ÷ Positive = NEGATIVE answer

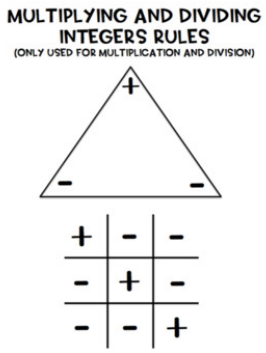
} If the integers have DIFFERENT SIGNS...answer is NEGATIVE

Example:

Divide the following.

a) $(+15) \div (+3)$

b) $28 \div (-4)$



c) $(-16) \div (+2)$

d) $(-15) \div (-3)$

e) $(-12) \div (-2) \div 3$

f) $150 \div (-3) \div (-5) \div 2$

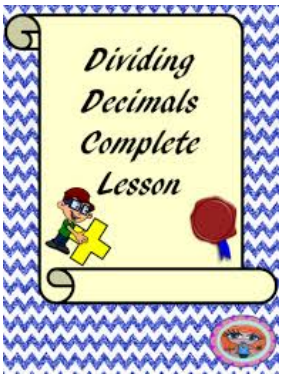
Note: Remember that when you have more than 2 integers, you have to work with 2 integers at a time!!

When Multiplying OR DIVIDING more than 2 Integers, simply count HOW MANY NEGATIVE integers there are. Then, apply these rules:

EVEN amount of NEGATIVE integers = POSITIVE answer

ODD amount of NEGATIVE integers = NEGATIVE answer

Decimal Rules for Dividing



We follow the same rules as the rules for integers.

Example:

Divide the following.

a) $(-2.73) \div (2.1)$

b) $(+2.28) \div (1.2)$

c) $(-18.2) \div (-6.5)$

d) $(-22.63) \div (3.1)$

e) $(4.08) \div (-1.7)$

f) $(-22.05) \div 6.3$

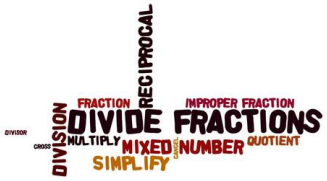
g) $(-12.71) \div (-3.1)$

h) $(0.5561) \div (-0.83)$

i) $-1.875 \div 1.25$

j) $-0.364375 \div 0.625$

Fraction Rules for Dividing



- To divide fractions we **don't** need a common denominator.
- We multiply by the **reciprocal.**
- Reduce to lowest terms, if possible.

What is a Reciprocal?

The **reciprocal** of a **fraction** is the found by "**flipping**" it so the numerator and denominator are swapped.

Multiplying a **fraction** by its **reciprocal** always gives you **one.**

$\frac{3}{7} \times \frac{7}{3} =$

Dividing Fractions

1. Keep the first fraction the same.
2. Change the division sign to a multiplication sign.
3. Flip the second fraction over to write the reciprocal.

Keep, Change, Flip

$$\frac{a}{b} \div \frac{c}{d}$$

$$\frac{a}{b} \times \frac{d}{c}$$

Example:

Divide the following.

a) $\frac{-2}{5} \div \frac{3}{10}$

b) $\frac{3}{4} \div -\frac{9}{8}$

c) $\frac{-2}{9} \div -\frac{4}{7}$

d) $2\frac{1}{2} \div \frac{25}{14}$

e) $\frac{8}{11} \div -4$

f) $16 \div \frac{-4}{5}$

Example:

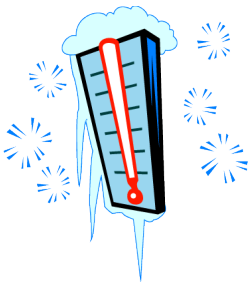
Complete these multiplication and division word problems. Show your work!

a) A plane seats 480 people. If the plane is $\frac{3}{4}$ full, how many people are on board the plane?

b) If a car travels 12.5 km on 1 liter of fuel, how many liters of fuel does it take to travel 100 km?

c) There are 30 people in a row at the movies. How many people are in $5\frac{1}{2}$ rows?

d) The temperature drops 10.5^0C over a 6 hour period. What was the hourly drop in temperature, assuming the temperature dropped the same amount each hour.

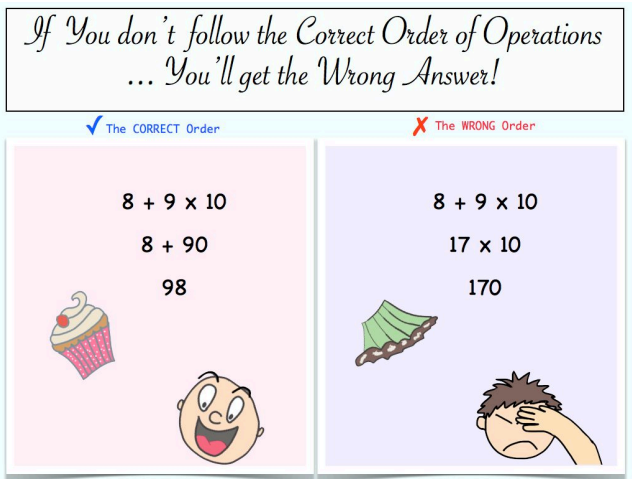


e) Suppose you find $\frac{1}{3}$ of a pizza in the fridge and you eat $\frac{1}{2}$ of it. What fraction of the whole pizza did you eat?

f) A tub contains 2.3 L of ice cream. It is shared equally among 5 people, how much does each person get?

g) A room measures 2.3 m by 3.4 m. If carpet cost \$18.25/m^2, calculate the cost to carpet the room before taxes.

Section 3.6 – Order of Operations with Rational Numbers



To avoid getting different answers when we evaluate an expression in math we use the following order of operations:

- Do the operations in the brackets first
- Evaluate the powers
- Multiply and divide, in order, from left to right
- Add and subtract, in order, from left to right

We can use the word **BEDMAS** to help us remember the order of operations.

	B	**Brackets**
	E	**Exponents**
whichever comes first	**D**	**Division**
	M	**Multiplication**
whichever comes first	**A**	**Addition**
	S	**Subtraction**

Example:

Calculate each of the following.

a) $(-2.4)\div 1.2-7\times 0.2$

b) $(-3.4+0.6)+4^2\times 0.2$

c) $\left(-\frac{2}{3}\right)\times\frac{1}{6}+\frac{1}{2}$

d) $\left(\frac{3}{4}-\frac{7}{8}\right)\div\left(-\frac{5}{16}\right)$

e) $\frac{7}{10}-\frac{1}{2}\times\frac{2}{5}$

f) $\frac{1}{3}-\frac{1}{2}\left(-\frac{1}{3}\right)^2$

g) $\left(2\frac{1}{3}\right)+\left(1\frac{1}{4}\right)\times\left(-\frac{2}{3}\right)$

h) $\frac{1}{4}-3\left(\frac{2}{3}+4\right)$

i) $\frac{10-7}{-4+2}$

j) $\left(-\frac{2}{3}+\frac{1}{2}\right)\times\left(\frac{-3}{2}\right)^2$

k) $\left[\frac{\frac{1}{4}-\left(-\frac{2}{5}\right)}{\frac{3}{5}-1\frac{1}{2}}\right]\times(-2)^2\div 3$

Example: (Error Questions)

a) A student's solution to a problem, to the nearest hundredth, is shown below. The solution is incorrect. Identify the errors and provide a correct solution.

Correct Solution:

$$\begin{aligned}&(-8.2)^2\div(-0.2)-2.9\times(-5.7)\\&=67.24\div(-0.2)-2.9\times(-5.7)\\&=67.24\div(-0.2)-16.53\\&=67.24\div(16.73)\\&\doteq 4.02\end{aligned}$$

b) Two students were asked to evaluate:

$$(-8)-2\left(24\div(-8)\right)^2$$

Why did both these students get the incorrect answer?
What is the correct answer?

Here are there calculations.

Student 1

$$\begin{aligned}&(-8)-2(24\div(-8))^2\\&=(-10)(24\div(-8))^2\\&=(-10)(-3)^2\\&=(-10)(9)\\&=-90\end{aligned}$$

Student 2

$$\begin{aligned}&(-8)-2(24\div(-8))^2\\&=(-8)-2(-3)^2\\&=(-8)-(-6)^2\\&=-8-36\\&=-44\end{aligned}$$

Correct Answer:

c) The following test questions was marked out of 3. What mark would you give this student? Justify your answer.

Calculate: $\frac{-7}{8} - \frac{3}{4} \div \frac{1}{5} - \frac{1}{4}$

Student's answer:

Correct Solution

$$\frac{-7}{8} - \frac{3}{4} \div \frac{1}{5} - \frac{1}{4}$$
$$= \frac{-7}{8} - \frac{3}{20} - \frac{1}{4}$$
$$= \frac{-7}{40} - \frac{3}{40} - \frac{1}{40}$$
$$= \frac{-11}{40}$$